Test 2
2002/2003
Math102

Question 1 :

(a) Find $\frac{dy}{dx}$:

(i) $y = \tanh^{-1}\left(\frac{1}{\coth x}\right)$ (ii) $y = e^x \sinh e^x$

(b) Show that

$$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right), \ |x| > 1$$

Question 2 :

Find the following limit if they exist:

(a) $\lim_{x\to 0} \frac{x - \tan^{-1} x}{x^3}$

(b) $\lim_{x\to(\pi/2)^-} (\tan x)^{\cos x}$

(c) $\lim_{x\to 0^+} \left(\frac{1}{\sin x} - \frac{1}{x}\right)$

Question 3 :

Find the surface area of a right circular cone of altitude 4cm and base radius *r* cm.

Question 4 :

Find the volume of the solid generated by revolving the region bounded by the graphs of $y = x^2$ and $y - 2x - 3 = 0$ about

(a) the *x*-axis.
(b) the line *x=3*.
(c) the line *y=9*.

Question 5 :

Evaluate each of the following integrals

(a) $\int \frac{x^3}{\sqrt{1+4x^2}}\, dx$ (b) $\int x\,(\ln x)^2\, dx$

(c) $\int e^{\sqrt{x}}\, dx$ (d) $\int \sec^{-1} x\, dx$