University of Bahrain
College of Science
Mathematics department
First Semester 2003-2004

Final Examination

Math 211 **Max. Mark: 50**
Date: 18 / 01 / 2004 **Time: 2 hours**

Question 1: **[5 marks]**

Let t be a real number. Discuss the rank and the nullity of the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 0 \\ t & 1 & 1 \\ 1 & t & 2 \end{bmatrix}$$

Question 2: **[3 × 4 marks]**

Let $V = \mathrm{IR}^3$ be an inner product space with the following weighted inner product:

If $u = (u_1, u_2, u_3)$ and $v = (v_1, v_2, v_3)$, then $\prec u, v \succ \; = \; \frac{1}{p}u_1v_1 + \frac{1}{p}u_2v_2 + \frac{1}{p}u_3v_3$,

where p is a fixed positive real number.

a) Find the angle θ between the vectors $u = (1, -1, 1)$ and $v = (3, 0, 6)$.

b) Find k such that the vectors $u_1 = (3, 2k, 9)$ and $v_1 = (k^2, 3, -1)$ are orthogonal.

c) Find a basis of $W^{\perp}$, where W is the subspace of V spanned by the vector $w = (3, 2, 1)$.

d) Find two vectors of norm 1 that are orthogonal to the given vectors $u_2 = (1, 1, 1)$ and $v_2 = (0, 1, 1)$.

Question 3: **[3 × 4 marks]**

In the vector space $V = \mathrm{P}_2$, consider $B = \{1, X\}$ and $B' = \{p = 1 - X \;,\; q = 2 - 3X\}$

a) Prove that B' is a basis of V.

b) Find the transition matrix from B' to B.

c) Find the transition matrix from B to B'.

d) Find the coordinates of $2 + X$ with respect to the basis B'.

<u>Question 4</u>: **[3 × 3 marks]**

Consider the following matrix

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

a) Find the eigenvalues of A.

b) Find a basis for each eigenspace of *A*.

c) Is there an invertible matrix *P* such that $P^{-1}AP$ is a diagonal matrix? Explain**.**

<u>Question 5</u>: **[3 × 4 marks]**

Let $T : \mathrm{IR}^2 \to \mathrm{IR}^3$ be the function defined by $T(x\,,\,y) = (x + y\,,\,2x\,,\,3y)$.

a) Show that *T* is a linear transformation.

b) Find the matrix of *T* with respect to the standard bases.

c) Find the kernel of *T*.

d) Find the range of *T*.