University of Bahrain
College of Science
Mathematics department
First Semester 2006-2007

Final Examination

Math 211
Duration: 2 hours
Date: 22 / 01 / 2007
Max. Mark: 50

Name:	
ID Number:	**Section:**

Instructions:

1) **Please check that this test has 5 questions and 7 pages.**
2) **Write your name, student number, and section in the above box.**

Marking Scheme

Questions	Max. Mark	Mark. Obtained
1	10	
2	9	
3	9	
4	8	
5	14	
Total	50	

Good Luck

Question 1: **[10 marks]**

Fill in the blanks to make a correct statement:

a) In any vector space V, if $ku = 0$, then ____

b) In a vector space V if $w = u - 2v$, then $\{u , v , w\}$ is ______________________________

c) If $S = \{p_1 , p_2 , p_3\}$ is linearly independent in P_2, then ______________________________

d) If $R^3 = \text{Span}\{ (2 , 3 , t) , (1 , 1 , 2) , (1 , 0, 0) \}$, then $t \neq$__________________________

e) If A is 2×2 invertible matrix and $m > 0$ such that $\det[(mAA^T)(mA)^{-2}] = 4$, then

$m =$____________

f) If A is a 3×2 matrix and B is a 3×3 matrix, then the size of $(AA^T + B)A^T$ is __________

g) If A is a 5×4 matrix, then the largest possible value of rank(A) is___________________

h) Suppose that $(p)_B = (-1 , -2 , 3)$ where $B = \{1 + x , -x^2 , x + x^2\}$ is a basis of P_2, then

$p =$ ____.

i) If $\det(\lambda I - A) = \lambda^5 - \lambda^4 + \lambda^3 - 5$, then $\det(A) =$ __________________________________

j) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 2 & 2 & 2 \end{bmatrix}$, then a basis for the column space of A is ______________________

<u>Question 2:</u> [3 + 3 + 3 marks]

a) Prove that if A and B are two matrices of size 2×2, then

$$\text{tr}(AB) = \text{tr}(BA) \qquad \text{and} \qquad \det(AB) = \det(BA)\,.$$

b) Let A and B be two matrices of size 2×2. If $AB = \begin{bmatrix} x & 2 \\ 8 & y \end{bmatrix}$ and $BA = \begin{bmatrix} 3 & 4 \\ 13 & 12 \end{bmatrix}$, find x and y.

c) Prove that $H = \{A \in \mathrm{M}_{22} : \text{tr}(A) = 0\}$ is a subspace of M_{22} and find its dimension.

(**Hint:** If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\text{tr}(A) = a + d$)

<u>Question 3</u>: **[3 + 3 + 3 marks]**

Let V be a vector space of dimension 4, and $B = \{u_1 , u_2 , u_3 , u_4\}$ be a basis of V. Define $u = u_1 + u_2$; $v = u_2 + u_3$; $w = u_3 + u_4$.

a) Prove that $S = \{u , v , w , u_4 \}$ is a basis of V.

b) Find the coordinates of the vector u_3 relative to the basis S.

c) Is $\{u , v , 2w , v - w\}$ a basis of V?

<u>Question 4</u>: **[4 + 4 marks]**

Let H = Span$\{v_1 , v_2 , v_3 , v_4\}$, where $v_1 = (1,1,1)$; $v_2 = (1,s,1)$; $v_3 = (s,1,1)$; $v_4 = (1,2,1)$.

a) Discuss dim$H$ according to the value of s, and find a basis of each case.

b) For $s = 3$, find a basis S of H consisting of vectors from v_1 , v_2 , v_3 , v_4. Then write the vectors not in the basis S as linear combination of those in S.

<u>Question 5</u>: **[3 + 4 + 3 + 4 marks]**

Consider the following matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & -2 & 3 \end{bmatrix}$

a) Find the eigenvalues of A.

b) Find a basis for each eigenspace of A.

c) Is there an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix? Explain.

d) Find A^n for every positive integer n.