University of Bahrain
College of Science
Mathematics department
Second Semester 2006-2007

Final Examination

Math 211
Duration: 2 hours
Date: 14 / 06 / 2007
Max. Mark: 50

Name:	
ID Number:	**Section:**

Instructions:

1) **Please check that this test has 5 questions and 7 pages.**
2) **Write your name, student number, and section in the above box.**

Marking Scheme

Questions	Max. Mark	Mark. Obtained
1	15	
2	9	
3	6	
4	8	
5	12	
Total	50	

Good Luck

Question 1: **[1.5 × 10 marks]**

Answer each of the following statements by **true** or **false**, then **justify** briefly your answers:

a) In any vector space V, if $au = bu$, then $a = b$.

b) In a vector space V, if $w \in \text{Span}\{u , v\}$, then $\{u , v , w\}$ is linearly dependent.

c) If A is a $n \times 2n$ matrix such that rank(A) = n, then nullity(A^T) = 0.

d) If $t = 0$, then $S = \{(1 , 2 , 1) , (-1 , 1 , 0) , (1 , -1, t)\}$ is a basis of $\mathbf{R}^3$.

e) If A and B are $n \times n$ matrices and $\det(A) = \det(B) = 1$, then $\det[2(AA^T)(2B)^{-2}] = 2^{-n}$.

f) If A is a 3×4 matrix and B is a 4×3 matrix, then the size of $(AA^T)^2 + (AB)^2$ is 3×3.

g) If A is a 6×4 matrix and nullity(A) = 0, then nullity(A^T) = 2.

h) If $A = \begin{bmatrix} \cos(x) & \sin(x) \\ -\sin(x) & \cos(x) \end{bmatrix}$, then $\lambda = 0$ is an eigenvalue of A.

i) If the characteristic polynomial of A is $P_A = (\lambda - 1)^5(\lambda - 2)^4(\lambda + 5)^2$, then det($A$) = 400.

j) If $A = \begin{bmatrix} a & 1 & 0 \\ 0 & a & 1 \\ 0 & 0 & a \end{bmatrix}$, then rank($A$) = 1.

Question 2: [5 + 4 marks]

a) Prove that $H = \{M \in \mathrm{M}_{22} : M^T = - M\}$ is a subspace of M_{22} and find its dimension.

b) Find a diagonal matrix X such that $AX^2 + BX + C = O$, where

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -2 \\ -1 & 0 \end{bmatrix}, \quad O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Question 3: **[3 + 3 marks]**

a) Solve $\begin{vmatrix} 1+x & x & 0 \\ 1 & 1+x & x \\ 0 & 1 & 1+x \end{vmatrix} = 0.$

b) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & a & 1 \\ a & 1 & 0 \end{bmatrix}$.

<u>Question 4</u>: **[2 + 3 + 3 marks]**

Consider the bases $B = \{u_1 , u_2\}$ and $B' = \{v_1 , v_2\}$ of the real vector space $\mathbf{R}^2$, where

$$u_1 = (0 , 1) \quad , \quad u_2 = (1 , 1) \quad , \quad v_1 = (1 , 0) \quad , \quad v_2 = (-1 , 2)$$

(i) Find the transition matrix from B' to B.

(ii) Find the transition matrix from B to B'.

(iii) Find the coordinate vector of $w = (2, 4)$ relative to the basis B'.

Question 5: **[3 + 3 + 3 + 3 marks]**

Consider the following matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$

a) Find the eigenvalues of A. Is A diagonalizable?

b) Find a basis for each eigenspace of A.

c) Find A^n for every positive integer n.

d) Let u_n and v_n be two sequences such that $u_o = 0$ and $v_o = 1$, and

$$u_n = 3\, u_{n-1} + 2\, v_{n-1}$$

$$v_n = 1\, u_{n-1} + 2\, v_{n-1}$$

Let $X_n = \begin{bmatrix} u_n \\ v_n \end{bmatrix}$, prove that $X_n = A\, X_{n-1}$ and $X_n = A^n X_o$, then find u_n and v_n .
