University of Bahrain
College of Science
Mathematics department
Second Semester 2007-2008

Final Examination

Math 211
Duration: 2 hours
Date: 16 / 06 / 2007
Max. Mark: 50

Name:	
ID Number:	**Section:**

Instructions:

1) **Please check that this test has 6 questions and 7 pages.**
2) **Write your name, student number, and section in the above box.**

Marking Scheme

Questions	Max. Mark	Mark. Obtained
1	9	
2	8	
3	9	
4	9	
5	8	
6	7	
Total	50	

Good Luck

<u>Question 1:</u> **[9 marks]**

a) Determine the determinant of a 3×3 matrix A such that $A^{-1} = 2A^T$.

b) Find the rank of a matrix A of size $(n+1) \times n$ such that $\text{Nullity}(A^T) = 2\,\text{Nullity}(A)$.

d) Let V be an inner product space V and u , v , w three nonzero vectors of V. Prove that, if $\langle u , v\rangle = \langle u , w\rangle = \langle v , w\rangle = 0$, then $\{u , v , w\}$ is linearly independent.

<u>Question 2:</u> **[4 + 4 marks]**

a) Prove that $H = \{\, p \in P_2 \;:\; p(1) + p'(1) = 0 \}$ is a subspace of P_2 and find its dimension.

b) Let A be a 2×2 matrix whose eigenvalues are 1 and -1. Show that there is a 2×2 invertible matrix P such that $P^{-1}AP = D$ is a diagonal matrix. Then prove that $A^{-1} = A$.

Question 3 **[9 marks]**

Let $V = \text{Span}\{f_1, f_2, f_3\}$, where $f_1 = 1$, $f_2 = e^x$, $f_3 = x\,e^x$.

a) Prove that $S = \{f_1, f_2, f_3\}$ is a basis of V.

b) Find the coordinates of $4 + (2 - 3x)\,e^x$ with respect to S.

c) Is $\{f_1, f_2, f_3, 1 + e^x\}$ a linearly independent set of V?

Question 4 **[6 + 3 marks]**

1) Let V be an inner product space and u , v are two nonzero vectors of V such that $\| u \| = \| v \| = h$. Let θ be the angle between u and v, and $w = \frac{1}{2}(u + v)$. Then

a) Find $< u , w >$ and $\| w \|$ as a function of h and θ .

b) Find the angle θ' between u and w as a function of θ .

2) Let A be a square matrix such that $A^2 = A$. Show that 0 and 1 are only the possible eigenvalues of A.

<u>Question 5</u> [8 marks]

Let W be the subspace of $\mathbf{R}^4$ generated by $u = (1 , 2 , 3)$ and $v = (2 , 4 , 2)$.

a) Find a basis of the orthogonal complement $W^{\perp}$ of W.

b) Find two vectors of norm 1 that are orthogonal to *u* and *v*.

Question 6 **[7 marks]**

Let $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & a \end{bmatrix}$. Find the eigenvalues of A and discuss whether A is diagonalizable as a varies.