University of Bahrain
College of Science
Mathematics department
Summer Semester 2004

Final Examination

Math 211 — **Max. Mark: 50**
Date: 28 / 08 / 2004 — **Time: 2 hours**

Question 1: **[4 + 4 + (3 × 3) marks]**

a) Let t be a real number. Discuss the rank and the nullity of the following matrix

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & t & 2 \\ t & 1 & 1 \end{bmatrix}.$$

b) For what value of a are the polynomials

$$p_1 = 1 + 2x + x^2 \quad , \quad p_2 = 1 + x^2 \quad , \quad p_3 = 1 + x + a\,x^2$$

linearly independent in P_2.

c) Let $V = \text{Span}\{u_1 , u_2 , u_3\}$, where

$$u_1 = (1, 0, 1, 0) \quad , \quad u_2 = (1, 1, 0, 0) \quad , \quad u_3 = (0, 1, 1, 0)$$

(i) Show that $S = \{u_1 , u_2 , u_3\}$ is a basis of V.

(ii) Find the coordinates of $v = (1, 2, 1, 0)$ relative to S.

(iii) Is the set $\{v , 3u_1 , 2u_2 , u_2 + u_3\}$ linearly independent?

Question 2: **[(3 × 3) + (3 × 3) marks]**

a) Consider the bases $B = \{u_1 , u_2\}$ and $B' = \{v_1 , v_2\}$ of the real vector space IR^2, where

$$u_1 = (1, 1) \quad , \quad u_2 = (1, 0) \quad , \quad v_1 = (0, 1) \quad , \quad v_2 = (-1, 2)$$

(i) Find the transition matrix from B' to B.

(ii) Find the transition matrix from B to B'.

(iii) Find the coordinates of $w = (3, 4)$ relative to the basis B'.

b) Let V be an inner product space and u , v are two nonzero vectors of V such that $\| u \| = \| v \| = h$. Let θ be the angle between u and v, and $w = \frac{1}{2}(u+v)$. Then

(i) Find $< u , w >$ as a function of h and θ.

(ii) Find $\| w \|$ as a function of h and θ.

(iii) Find the angle θ' between u and w as a function of θ.

Question 3: [3 × 5 marks]

Consider the following matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 4 & 1 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

a) Find the eigenvalues of *A*.

b) Find a basis for each eigenspace of *A*.

c) Is there an invertible matrix *P* such that $P^{-1}AP$ is a diagonal matrix? Explain**.**

d) Find P^{-1}.

e) Find A^n for every positive integer *n*.