University of Bahrain
College of Science
Mathematics department
Second Semester 2001-2002

<u>Final Examination</u>

Math 253 | **Max. Mark: 50**
Duration: 2 hours | **Date: 12th Jun, 2002**

<u>Question 1:</u> [8 marks]

a) Find a counterexample to show that the following statement is not a tautology

$$p \wedge q \wedge r \;\Rightarrow\; \neg[\,(\,p \Rightarrow q\,) \Rightarrow r\,]$$

b) Premises: $S \vee P$, $E \Rightarrow \neg P$, $S \vee P \Rightarrow (\neg P \Rightarrow S)$
Prove : $E \Rightarrow S$

Question 2: **[10 marks]**

a) Let n be a positive integer. Prove that

n is even if and only if 4 divides n^2.

b) Let $a \geq 2$ be a real number. Use mathematical induction to show that

$a^{n-1} \leq a^n - 1$ for $n = 1, 2, \ldots$

c) Let a be a positive real number. Prove by contradiction, that

if $a > 1$, then $a > \sqrt{a}$.

Question 3: [8 marks]

a) Prove that if $C \neq \emptyset$, then

$$A \cap B = \emptyset \quad \text{if and only if} \quad (A \times C) \cap (B \times C) = \emptyset$$

b) $(A' \cup B') \cap (A \cup B) = \emptyset$.

Use a Venn diagram to determine whether this conjecture is true or false. In case it is true, prove it. In case it is false, give a counterexample.

Question 4: **[12 marks]**

a) Let $f : \text{IR} \rightarrow \text{IR}$ be a function defined as $f(x) = 2x^2 + 1$. Find
$f[A]$, $f^{-1}[B]$, where $A = [1, 2]$ and $B = (-1, 1]$.

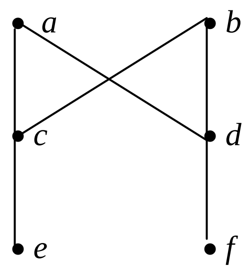
b) Let $f : \text{IN} \rightarrow \text{IR}$ be the function defined as $f(x) = 1 + x$, and $g : [1, \infty) \rightarrow \text{IR}$ be the function defined as $g(x) = \sqrt{x}$. Show that $g \circ f$ is well defined and find it explicitly.

c) Let $f : D \rightarrow C$ be a function and A a subset of D . Prove that if f is one-to-one, then
$f[D - A] \subseteq C - f[A]$

<u>Question 5:</u> [12 marks]

a) Let S be a relation on IR, defined as $S = \{ (x , y) \;:\; | x | + | y | \leq 1 \}$. Determine whether S is reflexive, symmetric, anti-symmetric and transitive. Is S an equivalence relation.

b) Let $A = \{ a , b , c , d , e , f \}$ and let the partial order $\leq$ be defined by the Hasse diagram :



(i) Find any maximal or minimal elements of A.

(ii) Find any greatest or least element of A.

(iii) Find lower bounds and upper bounds of $\{c , d , e , f \}$