University of Bahrain
College of Science
Mathematics department
Second Semester 2002-2003

Final Examination

Math 253 **Max. Mark: 50**
Duration: 2 hours **Date: th Jun, 2002**

Question 1: [8 marks]

a) Determine whether the following is a tautology

$[\ r \Rightarrow (\neg\, p \Rightarrow q\ )] \Leftrightarrow [(r \wedge \neg p) \Rightarrow q]$

b) Premises:

Prove :

Question 2: [10 marks]

a) Use mathematical induction to show that

$1\ 1! + 2\ 2! + \ldots + n\ n! = (n + 1)! - 1$ for $n = 1, 2, \ldots$

b) Prove or disprove: If x is real number, then $\sqrt{x^2+1} \geq \frac{|x|+1}{\sqrt{2}}$

c) Prove that there is a real number $\delta > 0$ such that for any $a > 0$, we have $a < \sqrt{x} < a + \frac{1}{a}$ whenever $a^2 < x < a^2 + \delta$.

Question 3: [4 + 4 + 4 marks]

a) Prove that $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

b) $A \cap B \subseteq (A \cap C') \cup (B \cap C)$

Use a Venn diagram to determine whether this conjecture is true or false. In case it is true, prove it. In case it is false, give a counterexample.

c) Write X in terms of A, B, and C, and the operations $\cap$, $\cup$, and $'$, where

$X = \{ x : x \notin A \Rightarrow (x \in B \Rightarrow x \in C) \}$

Question 4: **[(4 + 2) + 6 marks]**

a) Let f : IR-{0} $\to$ IR be a function defined as $f(x) = 1 + \frac{a}{x}$, where a > 0.

(i) Find $f[A]$, $f^{-1}[B]$, where $A = (0 , 1]$ and $B = \{-1,2\}$.

(ii) Is $f \circ f$ well defined ?

c) Let $f : D \to C$ be a function and A, B be two subset of D . Prove that if f is one-to-one, then: $A \cap B = \varnothing$ if and only if $f[A] \cap f[B] = \varnothing$.

Question 5: **[12 marks]**

Let R be a relation on IR, defined as $x\, S\, y \Leftrightarrow x + y$ is even.

a) Prove that R is an equivalence relation.

b) Find its equivalence classes.