University of Bahrain
College of Science
Mathematics department
Second Semester 2004-2005

Final Examination

Math 253
Duration: 2 hours
Date: 15th June, 2005
Max. Mark: 50

Name:	**I.D.No:**	**Section:**

Marking Scheme

Questions	Max. Mark	Mark. Obtained
1	10	
2	8	
3	8	
4	10	
5	6	
6	8	
Total	50	

<u>Question 1:</u> **[10 marks]**

1) Let p , q and n be positive integers. Consider the statement:

$$p + q = n \Rightarrow \forall \varepsilon \in (0 , 1) , \big(p \geq \varepsilon n \quad \vee \quad q \geq (1- \varepsilon)n \big) \qquad (*)$$

- **a)** Write the negation of (*).
- **b)** Write the contrapositive of (*)
- **c)** Prove (*)

2) Prove that $(p \wedge q) \Leftrightarrow r \equiv [(p \wedge q \wedge r) \vee (\neg p \wedge \neg r)] \vee (\neg q \wedge \neg r)$.

<u>Question 2:</u> [8 marks]

1) Prove or disprove in **Z**: $\exists\, x, y, z,\ (x, y, z \text{ are odd } \wedge\ x + y + z = 100)$.

2) Prove that 3 divides $n^3 - n$, for every $n = 0, 1, 2, \ldots$

<u>Question 3:</u> [8 marks]

1) Prove that if $A \cap B = \varnothing$, then $\mathrm{P}(A) \cap \mathrm{P}(B) = \{\varnothing\}$.

2) Let A a non-empty set**.** Prove that if $(A - B) \times A = \varnothing$, then $A \subseteq B$.

<u>Question 4:</u> [10 marks]

1) Let $f : (0 , \infty) \to \mathbf{R}$ be the function defined by $f(x) = 1 + \dfrac{1}{x}$.

a) Find $f[A]$, where $A = [1, 2]$.

b) Find $f^{-1}[B]$, where $B = (2 , 5)$.

c) Let $g : \mathbf{N} \to \mathbf{Z}$ be the function defined by $g(x) = 2x - 1$. Is $f \circ g$ well defined?

2) Let $f : D \to D$ be a function such that $f \circ f = 1_D$. Prove that f is bijective.

Question 5: **[6 marks]**

Let $f : D \to C$ be a function and A , B be two subsets of D.

1) Prove that $f[A] - f[B] \subseteq f[A - B]$.

2) Prove that if f is one-to-one, then $f[A - B] \subseteq f[A] - f[B]$.

<u>Question 6:</u> [8 marks]

Let R be a relation on a subset A of $(0\ ,\ \infty)$, defined by: $x\ R\ y \iff x\ \mathrm{Ln}(y) = y\ \mathrm{Ln}(x)$.

a) Prove that R is an equivalence relation.

b) Suppose that $A = \{1\ ,\ 2,\ 4\ ,\ 8\ ,\ 16\}$. Find the equivalence classes of R.