University of Bahrain
College of Science
Mathematics department
First Semester 2007-2008

Final Examination

Math 253
Duration: 2 hours
Date: 26th January, 2008
Max. Mark: 50

Name:	I.D.No:	Section:

Marking Scheme

Questions	Max. Mark	Mark. Obtained
1	8	
2	12	
3	11	
4	12	
5	7	
Total	50	

<u>Question 1:</u> [(2+3)+ 3 marks]

1) Consider the statement: $\exists \delta > 0 \ \left(\frac{1}{4} < x < 1 + \delta\right) \Rightarrow \left(\frac{1}{2} < \sqrt{x} < \frac{3}{2}\right).$ (*)

a) Write the negation of (*).

b) Prove (*).

2) Premises: $p \vee q$, $q \Rightarrow \neg (r \wedge s)$, $p \vee q \Rightarrow (\neg q \Rightarrow p)$

Prove : $(r \wedge s) \Rightarrow p$

<u>Question 2:</u> [4+4+4 marks]

a) Prove or disprove: If a is real number, then $\sqrt{4a^2+1} \geq \dfrac{2|a|+1}{\sqrt{2}}$.

b) Use a mathematical induction to show that:

$$2^n \text{ divides } (n+1)(n+2)\ldots(2n-1)(2n), \quad \text{for } n = 0, 1, 2, \ldots$$

c) Prove by contradiction: If $|x| < \varepsilon$ for all $\varepsilon > 0$, then $x = 0$.

<u>Question 3:</u> **[2 + 3 + 3 + 3 marks]**

Define $A + B = (A - B) \cup (B - A)$

- **a)** Draw Venn diagram for $(A + B) - C$.
- **b)** Prove that $A + B = (A \cup B) - (A \cap B)$.
- **c)** Prove that $A + (A \cap B) = A - B$.
- **d)** Use a pick-a-point to show that if $A + B \subseteq C$, then $A \cup C \subseteq B \cup C$.

Question 4: **[(3+3+3)+3 marks]**

1) Let $f : (0 , \infty) \to \mathbf{IR}$ be a function defined as $f(x) = x^2 + 2x$.

a) Find $f[A]$, $f^{-1}[B]$, where $A = [1 , 2]$ and $B = \{1 , -1\}$.

b) Is f onto? Is f one-to-one?

c) Show that $f \circ f$ is well defined and find it explicitly.

2) Let $f : D \to C$ be a function and A, B be two subset of D . Prove that if f is one-to-one, then: $A \cap B = \varnothing$ if and only if $f[A] \cap f[B] = \varnothing$.

Question 5: **[7 marks]**

Let R be a relation on $\mathbf{Z}$, defined as $x\,R\,y \Leftrightarrow x + y$ is even.

a) Prove that R is an equivalence relation.

b) Find its equivalence classes.
