

University of Bahrain  
College of Science  
Mathematics department  
Second Semester 2008-2009

**Final Examination**

Math 253  
Date: 16 / 06 / 2009

Max. Marks: 50  
Duration: 2 hours

Name:
ID Number:

**Instructions:**

- 1) Please check that this test has 6 questions and 8 pages.
- 2) Write your name, student number, and section in the above box.

Question	Max. Marks	Marks obtained
1	8	
2	8	
3	8	
4	8	
5	10	
6	8	
<b>Total</b>	<b>50</b>	

**Good Luck**

**Question 1: [4 + 4 marks ]**

a) Prove that the following statement is a tautology

$$(p \Leftrightarrow q) \Rightarrow [(p \wedge r) \Leftrightarrow (q \wedge r)]$$

and, give a counter-example to show that the converse is false.

b) Let  $(a_n)$  be a sequence such that  $a_1 = 1$ , and  $a_n = 3a_{n-1} - 1$  for  $n > 1$ . Prove that

$$a_n = \frac{3^{n-1} + 1}{2} \text{ for } n = 1, 2, 3, \dots$$

**Question 2 [ 8 marks]**

Let  $U = \{2, 3, \dots\}$ . Consider the predicate:

$$\exists m \in U, \exists n \in U, \left(\frac{1}{m} + \frac{1}{n} = \frac{7}{17}\right) \quad (*)$$

- a) Show that  $m \neq 2$  and  $n \neq 2$ .
- b) Show that  $m < 5$  or  $n < 5$ .
- c) Deduce that  $m$  or  $n$  must be 3 or 4.
- d) Derive the solution set of the predicate (\*).

**Question 3: [4 + 4 marks ]**

a) Simplify  $[(A - B)' - (B - A)] \cup (A \cap B)$ .

b) Prove that,  $A - B \subseteq C$  if and only if  $A - C \subseteq B$ .

**Question 4: [ 4 + 4 marks]**

a) Let  $A$  be a subset of  $X$ . Prove or disprove

$$(X \times X) - (A \times A) = [(X - A) \times X] \cup [X \times (X - A)].$$

b) Let  $f: D \rightarrow C$  be a function and  $X$  a subset of  $D$ . If  $f$  is bijective, show that

$$f[D - X] = C - f[X].$$

**Question 5: [10 marks]**

Let  $f: \mathbb{R} \rightarrow (-1, 1)$  be the function defined  $f(x) = \frac{x}{1 + |x|}$ .

- a) Prove that :  $ab > 0 \Leftrightarrow f(a)f(b) > 0$ , for every  $a, b \in \mathbb{R}$ .
- b) Prove that  $f$  is one to one.
- c) Prove that  $f$  is onto. (Distinguish the cases:  $y \geq 0$  and  $y < 0$ )
- d) Find  $f^{-1}(A)$ , where  $A = \{ \frac{1}{2}, -\frac{1}{4} \}$ .
- e) Determine the inverse function of  $f$ .



**Question 6: [4 + 4 marks]**

a) Let  $f$  is a real valued function. Let  $R$  be a relation defined on  $\mathbb{R}$  by

$$x R y \Leftrightarrow f(x) = f(y).$$

Show that  $R$  is an equivalence relation.

b) If  $f$  is defined by  $f(x) = x^2 - x + c$ , find the equivalence class of any element  $a$  of  $\mathbb{R}$ .

**Solution:**