University of Bahrain
College of Science
Mathematics department
First Semester 2009-2010

Final Examination

Math 253 | **Max. Marks: 50**
Date: 14 / 01 / 2010 | **Duration: 2 hours**

Name:	
ID Number:	

Instructions:

1) **Please check that this test has 6 questions and 8 pages.**
2) **Write your name, student number, and section in the above box.**

Question	Max. Marks	Marks obtained
1	7	
2	8	
3	8	
4	8	
5	11	
6	8	
Total	50	

Good Luck

Question 1: **[0.5× 14 = 7 marks]**

a) Answer each of the following statements by **true** or **false**

Statement	True	False
7 is odd if and only if 6 is even		
$0^2 = -1$ and $1 + 1 = 2$		
$x^2 \geq 0$ or $x^2 < 0$		
If $1 > 2$, then $\cos^2(1) + \sin^2(1) = 1$		
$\varnothing \in \{\{\varnothing\}\}$		
$\varnothing \subseteq \{\varnothing , \{\varnothing\}\}$		
$P(\{\varnothing\}) = \{\varnothing , \{\varnothing\}\}$		
$\exists x\ (x \in \{\varnothing\} \ \wedge\ x \subseteq \{\varnothing\})$		

b) Consider the following statement:

$$(x > 2 \ \wedge\ |x + y| = 2) \ \Rightarrow\ (y < 2 \vee\ y < -4).$$

Fill in the blanks:

converse	
negative	
contrapositive	

c) Consider the following statement: $r \ \Rightarrow\ \neg p \vee q \ \Leftrightarrow\ r \wedge p \ \Rightarrow\ q$

and suppose that p and q are true and r is false.

Fill in the blanks:

Put parentheses	
Main connective	
Truth value	

<u>Question 2</u> [4 + 4 marks]

a) Let $U = \mathbb{N}$. Consider the predicate:

$$\exists\, x \in U,\ \ \exists\, y \in U\,,\ (x^2 = 4y - 2) \qquad\qquad (*)$$

i) Show that x is even.

ii) Derive the solution set of the predicate (*).

b) Let (a_n) be a sequence such that $a_1 = 1$, and $a_n = \sqrt{(a_{n-1})^2 + \pi}$ for $n > 1$. Determine a_n for $n = 1, 2, 3, \ldots$

<u>Question 3:</u> [4 + 4 marks]

a) Use algebraic method to prove: $(A \cup B) - (A \cup C) = (B - C) - A$.

b) Let A, B and C be three sets such that $A \times B \subseteq A \times C$. Prove

(i) $A \times (B - C) = \varnothing$.

(ii) $A = \varnothing$ or $B \subseteq C$.

<u>Question 4:</u> [4 + 4 marks]

a) Let $f: D \to C$ be a one-to-one function, and A , B be two subsets of D. Prove that, if $A \cap B = \emptyset$, then $f[A] \cap f[B] = \emptyset$.

b) Let $f : D \to C$ be a function and Y a subset of C. Show that $f^{-1} [C - Y] = D - f^{-1} [Y]$.

Question 5: **[3+2+2+2+2 marks]**

Let $f : \mathbb{R}\text{-}\{0\} \to \mathbb{R}\text{-}\{1\}$ be a function defined as $f(x) = 1 + \dfrac{a}{x}$, where $a > 0$.

a) Prove that f is bijective.

b) Find the inverse function of f.

c) Find $f[A]$, where $A = (0\,,1]$.

d) Find $f^{-1}[B]$, where $B = \{-1,2\}$.

e) Is $f \circ f$ well defined ?

<u>Question 6:</u> **[4 + 4 marks]**

Let R be a relation on $\mathbb{Z}$, defined as $x\,R\,y \Leftrightarrow x + y$ is even.

- **a)** Prove that R is an equivalence relation.
- **b)** Find its equivalence classes.