University of Bahrain
Faculty of Science
Department of mathematics
First semester 2004-2005

Date: 11 / 01 / 2005 **-Maths 311-**
Time: 2 hours **Final Examination**

Question 1: [3+5+3+3 marks]:

Let G = (-1 , 1). Define on G the binary operation : $a * b = \dfrac{a+b}{1+ab}$

a) Show that $*$ is a binary operation.

b) Show that $(G , *)$ is an Abelian group.

c) Solve the equation $\dfrac{1}{2} * x = 1$.

d) Prove that no element of G is of order 3.

Questions 2: [6×4 marks]

Prove the following assertions:

a) $(\mathbb{Q}, +)$ is not cyclic.

b) $(\mathbb{Q}, +)$ is not isomorphic to $(\mathbb{Q}/\mathbb{Z} ,+)$.

c) If A and B are two subgroups of G such that $|A|$ is a prime number, then either $A \cap B = \{e\}$ or $A \subseteq B$.

d) If $f : G \to H$ and $g : G \to K$ are two onto homomorphisms such that Ker(f) = Ker(g), then $H \cong K$.

e) A group G of order p^r is not simple (p a prime number and $r > 1$).

f) A group G of order $(35)^2$ is not simple.

Question 3: [3×4 marks]

Let G be a group and a an element of G of order 3. Let $f : \mathbb{Z}_{12} \to G$ be the function defined by $f(i) = a^i$.

a) Prove that f is a homomorphism.

b) Find K = Ker(f) and the cosets of $\mathbb{Z}_{12} / K$.

c) Find $f(\mathbb{Z}_{12})$ and give the correspondence between $\mathbb{Z}_{12} /_K$ and $f(\mathbb{Z}_{12})$.