University of Bahrain
College of Science
Mathematics department
Second Semester 2006-2007

Final Examination

Math 311
Duration: 2 hours
Date: 23 / 06 / 2007
Max. Mark: 50

Name:	
ID Number:	**Section:**

Instructions:

1) **Please check that this test has 5 questions and 7 pages.**
2) **Write your name, student number, and section in the above box.**

Marking Scheme

Questions	Max. Mark	Mark. Obtained
1	10	
2	10	
3	10	
4	6	
5	14	
Total	50	

Good Luck

Question 1: **[2.5 × 4 marks]**

Find the answers to the following questions:

a) Let a be an element of a group G such that $o(a) = n$. If $n = km$, determine $o(a^k)$.

b) In a group G, if $x a = b$, $c = x a x^{-1}$ and $d = x b x^{-1}$, then determine $c d^{-1}$.

c) Find the order of $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

d) Let H and K be two cyclic groups such that $o(H) = 3$ and $o(K) = 5$. Find a group isomorphic to $H \oplus K$.

Question 2: **[2.5 × 4 marks]**

Find the answers to the following questions:

a) Let $a = (12 , 10 , 18) \in Z_{18} \oplus Z_{15} \oplus Z_{24}$. Find the order of a.

b) Let $\sigma , \rho \in S_n$. Prove that $\sigma^{-1}\rho^{-1}\sigma\rho \in A_n$.

c) Let H be a normal subgroup of G and a , b are two elements of G. Prove that, if $a\,b \in H$, then $b\,a \in H$.

d) Let H be a subgroup of G such that $(G : H) = 2$. Prove that $a^2 \in H$ for all $a \in G$.

Question 3: **[6 + 4 marks]**

Let $(G\ ,\ .)$ be a group and H be a subgroup of $(G\ ,\ .)$.

a) Show that $(G\ ,\ *)$ is a group for the following binary operation: $x * y = x\,a^{-1}\,y$.

b) Prove that $F = \{x\,a : x \in H\}$ is a subgroup of $(G\ ,\ *)$.

<u>Question 4</u>: **[4 + 4 marks]**

In a group G, let a and b be two elements such that $a\,b = b\,a$, $o(a) = 4$ and $o(b) = 5$.

Prove the following:

a) $< a > \cap < b > = \{e\}$.

b) $o(a\,b) = 20$.

<u>Question 5</u>: **[3 + 2 + 3 + 3 + 3 marks]**

Let G be an abelian group, and H and K be two subgroups of G. Define a function

$\psi : H \oplus K \to HK$ by $\varphi((x , y)) = x\, y^{-1}$

a) Prove that ψ is a homomorphism.

b) Show that ψ is onto.

c) Prove that $\text{Ker}(\psi) = \{(x , x) : x \in H \cap K \}$.

d) Prove that $\text{Ker}(\psi) \cong H \cap K$.

e) Deduce that if H and K are finite, then $| HK | = \dfrac{| H \oplus K |}{| H \cap K |}$.