University of Bahrain
College of Science
Mathematics department
Second Semester 2007-2008

Final Examination

Math 311
Duration: 2 hours
Date: 16 / 06 / 2007
Max. Mark: 50

Name:	
ID Number:	**Section:**

Instructions:

1) **Please check that this test has 5 questions and 7 pages.**
2) **Write your name, student number, and section in the above box.**

Marking Scheme

Questions	Max. Mark	Mark. Obtained
I	18	
II	6	
III	6	
IV	6	
V	14	
Total	50	

Good Luck

Question I: [18 marks]

a) Find the order of (3 , 10 , 5) in $\mathbf{Z}_6 \times \mathbf{Z}_{15} \times \mathbf{Z}_{10}$.

b) If G is a cyclic group of order 360, show that $G \cong \mathbf{Z}_8 \times \mathbf{Z}_9 \times \mathbf{Z}_5$.

c) List the elements of $\mathbf{Z}_4 \times \mathbf{Z}_3 / < (0,1) >$.

d) Is the multiplicative group $(\mathbf{R}^*, .)$ cyclic ?

e) Let a be an element of a multiplicative group G. If $o(a) = p$ is a prime number,

Find the order of a^{3p+1}.

f) Prove that K is a Sylow 2-subgroup of A_4, where

$$K = \{ \rho_o , \sigma_1 = (1\ 2)(3\ 4) , \sigma_2 = (1\ 3)(2\ 4) , \sigma_3 = (1\ 4)(2\ 3) \}$$

Question II: **[3 + 3 marks]**

Let p be a prime number and G be a finite group such that p divides $o(G)$.

a) Prove that a Sylow p-group H of G is normal if and only if H is the unique Sylow p-subgroup of G.

b) Conclude that a group G of order $p^2(p - 1)$ is not simple.

Question III: **[3 + 3 marks]**

If $f : G \to G'$ be a homomorphism and $a \in G$.

(1) If o(a) is finite, show that o(f(a)) divides o(a).

(2) Conclude that, if o(a) is a prime number, then o(a) = o(f(a)) or $a \in$ Ker(f).

<u>Question IV:</u> [3 + 3 marks]

Let G be an Abelian group of order $2n$, where n is an odd positive integer.

- **a)** By using Cauchy's Theorem, prove that G has an element a of order 2.
- **b)** Show that a is the unique element of G of order 2.

<u>Question V:</u> [14 marks]

Let $(G, .)$ be an **Abelian** group and n a fixed positive integer.

Consider the function $\varphi : G \times G \rightarrow G$

$(x, y) \rightarrow x^n y^{-1}$

a) Prove that φ is a homomorphism.

b) Prove that φ onto.

c) Show that $H = \{ (x, x^n) : x \in G \}$ is a normal subgroup of G.

d) What is $G \times G / H$?

e) If the order of G is finite, find $o(H)$.