

University of Bahrain
College of Science
Mathematics department
First Semester 2005

Final Examination

Math 352
Date: 31 / 12 / 2005

Max. Marks: 50
Duration: 2 hours

Name:
ID Number:

Instructions:

- 1) Please check that this test has 4 questions and 5 pages.
- 2) Write your name, student number, and section in the above box.

Question	Max. Marks	Marks obtained
1	14	
2	12	
3	12	
4	12	
Total	50	

Good Luck

Question 1: [7 + 7 marks]

a) Find the remainder when $5^{16n+3} + 2(14!)$ is divided by 17.

b) Assuming that 594 divides the integer 465X26Y4, find the digits X and Y.

Question 2: [6 + 6 marks]

Let a, b, c be three positive integers.

a) Prove that if $\gcd(a, b) = 2$ and $\gcd(a, c) = 3$ and a divides bc , then a divides 6.

b) Prove that if $\gcd(a, b) = 1$, then $\gcd(a + b, a^2 + ab + b^2) = 1$

Question 3: [6 + 6 marks]

a) Divide 200 into two summands such that one is divisible by 5 and the other by 13

b) Show that, for any integer a , the integer $a^2 + a + 5$ ends in one of the digits 1, 5, or 7.

Question 4: [6 + 6 marks]

a) Use Chinese Remainder Theorem to find an integer a such that

$$4 / a + 1, 9 / a + 2, 25 / a + 3$$

b) Let p be an odd prime number. Prove that $2p$ divides $a^{2p} - a^2 - a^p + a$ for any integer a .