University of Bahrain
College of Science
Mathematics department
First Semester 2008-2009

Final Examination

Math 352   **Max. Marks: 50**
Date: 13 / 01 / 2009   **Duration: 2 hours**

Name:	
ID Number:	

Instructions:

1) **Please check that this test has 6 questions and 7 pages.**

2) **Write your name, student number, and section in the above box.**

Question	Max. Marks	Marks obtained
1	10	
2	8	
3	8	
4	8	
5	8	
6	8	
Total	**50**	

Good Luck

Question 1: [5 + 5 marks]

a) Find the remainder when $7^{18n+3} + 4\,(16!)$ is divided by 19.

b) Use Chinese Reminder Theorem to determine an integer x having the remainder 2, 3, 4 when dividing by 3, 4, 5 respectively.

Question 2 [4 + 4 marks]

a) Show that $\varphi(n) = \dfrac{n}{2}$ if and only if $n = 2^k$ for some integer $k \geq 1$.

b) Prove that if $\gcd(a, b) = 1$, then $\gcd(a - b, a^2 - ab + b^2) = 1$

Question 3: [4 + 4 marks]

a) Let $p > 2$ be a prime number. Prove by induction that $m < p^{m-1}$ for $m = 2, 3, \ldots$

b) Use (a) to show that n divides $(n-1)!$.

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Question 4: **[4 + 4 marks]**

a) Let $n > 1$ be an integer not of the form $6k + 3$. Prove that $n^2 + 2^n$ is composite.

b) Find the units digit of $5^m + 6^n + 11^{n+m}$.

Question 5: **[4+ 4 marks]**

a) Let b be a positive integer . Show that if b has 0 for **units** digit, then

$$(1 + b)^n \equiv 1 + n\, b + \frac{n(n-1)}{2} b^2 \pmod{1000}$$

b) Use (a) to find the **first three digits** of $(131)^{412}$.

Question 6:   [4 + 4 marks]

a) Let p be a prime number and a an integer such that g.c.d$(a , p) = 1$.

Verify that $x_o = a^{p-2} b$ is a solution of the linear congruence $a\,x \equiv b \pmod{p}$.

b) Apply (a) to solve $8\,x \equiv 3 \pmod{31}$.