University of Bahrain
College of Science
Mathematics department
First Semester 2009-2010

Final Examination

Maths 352 **Max. Marks: 50**
Date: 17/ 01/ 2010 **Duration: 2 hours**

Name:
ID Number:

Instructions:

1) **Please check that this test has 6 questions and 8 pages.**
2) **Write your name, student number, and section in the above box.**

Question	Max. Marks	Marks obtained
1	8	
2	8	
3	8	
4	10	
5	8	
6	8	
Total	50	

Good Luck

Question 1: **[4+ 4 marks]**

Let f be a function defined by $f(k) = \binom{n+k}{k-1}$ for $k \geq 1$.

a) Prove that $f(k+1) - f(k) = \binom{n+k}{k}$.

b) Deduce that $\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} + \binom{n+3}{3} + \ldots + \binom{2n}{n} = \binom{2n+1}{n}$.

Question 2 [4 + 4 marks]

a) Let $a > 1$ be an integer. Prove that $a^{4n} + 4$ is composite.

b) If $\gcd(a, b) = 1$, prove that $\gcd(a^2 - b^2, a^3 + b^3) = |a + b|$.

<u>Question 3:</u> [4 + 4 marks]

a) Find an integer a such that $4 / a + 1$, $9 / a + 2$, $25 / a + 3$.

b) Prove that if the integer n has k distinct odd prime factors, then

$$\varphi(n) \equiv 0 \pmod{2^k}$$

Question 4: **[4 + 4 + 2 marks]**

a) Let a and b be two integers. Prove that $(a+b)^{17} \equiv a^{17} + b^{17} \pmod{17}$.

(Hint: show that 17 divides $\binom{17}{k}$ for every $0 < k < 17$)

b) Deduce that $1^{17} + 2^{17} + \ldots + (n)^{17} \equiv \left[\frac{n(n+1)}{2}\right]^{17} \pmod{17}$.

c) Find the remainder when $1^{17} + 2^{17} + \ldots + (1000)^{17}$ is divided by 17.

Question 5: [4+ 4 marks]

a) Given an integer N with n digits. Let M the integer formed by reversing the order of the digits. Show that $(N)^n - (M)^n$ is divisible by 11.

b) Show the equation $x^2 - 10y = 7$ has no solution in the set of integers.

<u>Question 6:</u> **[4 + 4 marks]**

a) Let p be an odd prime number and a an integer. Use Fermat's Theorem to prove that $2p$ divides $a^{2p} - a^{2} - a^{p} + a$.

b) Let p and $p + 2$ be two primes. Use Wilson's Theorem to prove that $p(p + 2)$ divides $4(p-1)! + p + 4$