**University of Bahrain**
**College of Science**
**Department of Mathematics**
**First Semester 2002/2003**
**Test 2**

**STAT 273** **Time** : 60 minutes
**Date:** 24/12/2002 **Max. Marks:** 25

**Question 1:**

Suppose that $X_1, X_2, ...., X_n$ constitutes a random sample of size n from a uniform population over $(0, \beta)$. Let

$\overline{X} = \dfrac{X_1 + X_2 + .... + X_n}{n}$ $\quad and \quad Y_n = Max.(X_1, X_2, ..., X_n)$

a) Using $\overline{X}$ *and* $Y_n$ find two unbiased estimators of $\beta$.
b) Compute the variances of your estimators.

**Question 2:**

Let $X_1, X_2, and\ X_3$ constitute a random sample of size $n = 3$ from a Bernoulli distribution. Further, Let $t = (2X_1 + X_2 + X_3)/4$

a) Is $t$ a sufficient estimator of the parameter $\theta$? Given reason.
b) Find the maximum likelihood estimator of $\theta$.

**Question 3:**

The following are the number of minutes it took a sample of 8 men and 6 women to complete an application form for a position.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Men : | 16.5, | 20.0, | 17.0, | 18.5, | 19.0, | 21.0, | 15.0, | 16.0 |
| Women: | 18.0, | 20.5 | 21.5, | 22.0, | 24.0, | 23.0 | | |

Suppose that the two sample are independently drawn from $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$ respectively.

a) If $\sigma_1^2 = \sigma_2^2 = \sigma_1^2$ obtain a 95% confidence interval for $(\mu_1 - \mu_2)$.
b) Obtain a 90% confidence interval for $(\sigma_1^2 / \sigma_2^2)$ assuming that $\sigma_1^2 \neq \sigma_2^2$.