Fin 220

Section : 4

Chapter 3

The Time Value of Money

©

Time Value of Money

- The most important concept in finance
- Used in nearly every financial decision
  - Business decisions
  - Personal finance decisions

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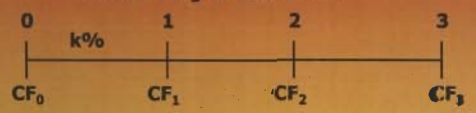
Cash Flow Time Lines

Graphical representations used to show timing of cash flows

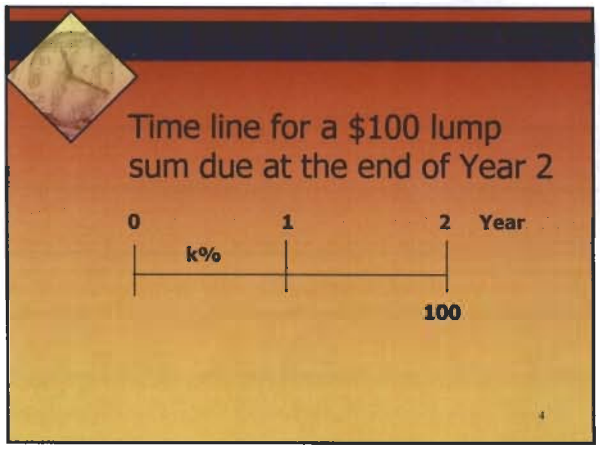


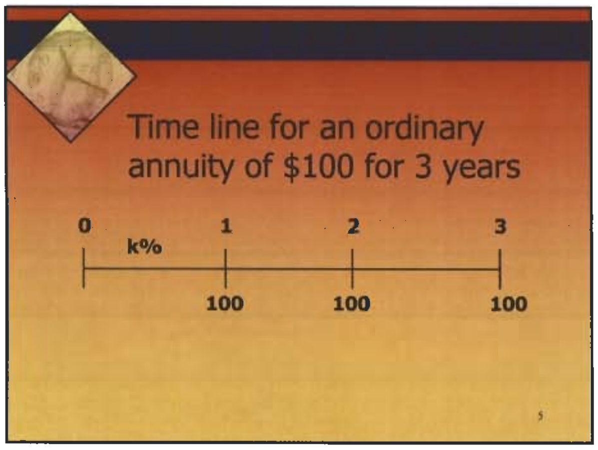
Time 0 is today

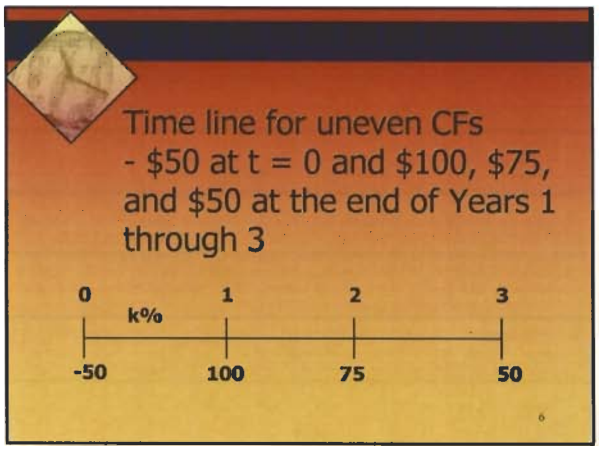
Time 1 is the end of Period 1 or the beginning of Period 2.

3

Dr. Nadhem Al-Saleh


Time line for a $100 lump sum due at the end of Year 2
0
1
2
Year
k%
100


Time line for an ordinary annuity of $100 for 3 years
0
1
2
3
k%
100
100
100


Time line for uneven CFs - $50 at t = 0 and $100, $75, and $50 at the end of Years 1 through 3
0
1
2
3
k%
-50
100
75
50

Future Value

The amount to which a cash flow or series of cash flows will grow over a period of time when compounded at a given interest rate.

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Future Value

How much would you have at the end of one year if you deposited $100 in a bank account that pays 5 percent interest each year?

$$FV_n = FV_1 = PV + INT$$
$$= PV + (PV \times k)$$
$$= PV(1 + k)$$
$$= \$100(1 + 0.05) = \$100(1.05) \boxed{= \$105}$$

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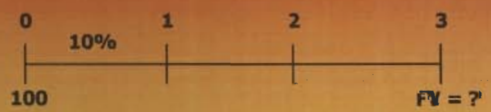
What's the FV of an initial $100 after 3 years if k = 10%?



Finding FV is Compounding.

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Future Value

After 1 year:

$$FV_1 = PV + Interest_1 = PV + PV(k)$$
$$= PV(1 + k)$$
$$= \$100\,(1.10)$$
$$= \$110.00.$$

After 2 years:

$$FV_2 = PV(1 + k)^2$$
$$= \$100\,(1.10)^2$$
$$= \$121.00.$$

After 3 years:

$$FV_3 = PV(1 + k)^3$$
$$= 100\,(1.10)^3$$
$$= \$133.10.$$

In general, $FV_n = PV(1 + k)^n$

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Three Ways to Solve Time Value of Money Problems

- Use Equations
- Use Financial Calculator
- Use Electronic Spreadsheet

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Numerical (Equation) Solution

Solve this equation by plugging in the appropriate values:

$$FV_n = PV(1+k)^n$$

PV = \$100, k = 10%, and n =3

$$FV_n = \$100(1.10)^3$$
$$= \$100(1.3310) = \$133.10$$

Financial Calculator Solution

Financial calculators solve this equation:

$$FV_n = PV(1+k)^n$$

There are 4 variables. If 3 are known, the calculator will solve for the 4th.

Financial Calculator Solution

Here's the setup to find FV:

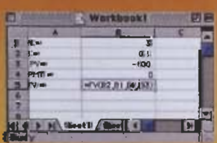
INPUTS	3	10	-100	0	?
	N	I/YR	PV	PMT	FV
OUTPUT					133.10

Clearing automatically sets everything to 0, but for safety enter PMT = 0.

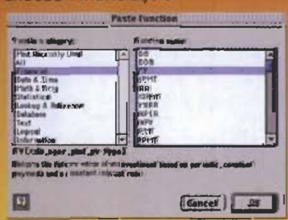
Set: P/YR = 1, END

Spreadsheet Solution

Set up Problem



Click on Function Wizard and choose Financial/FV



Spreadsheet Solution

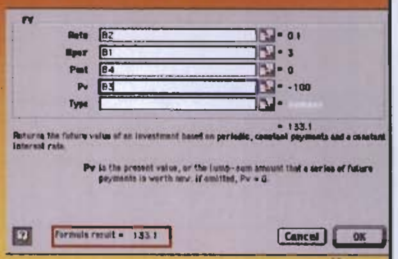
Reference cells:

Rate = interest rate, k

Nper = number of periods interest is earned

Pmt = periodic payment

PV = present value of the amount



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Present Value

- **Present value** is the value today of a future cash flow or series of cash flows.
- **Discounting** is the process of finding the present value of a future cash flow or series of future cash flows; it is the reverse of compounding.

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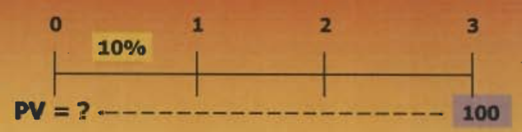
What is the PV of $100 due in 3 years if k = 10%?



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What is the PV of \$100 due in 3 years if k = 10%?

Solve $FV_n = PV(1 + k)^n$ for PV:

$$PV = \frac{FV_n}{(1+k)^n} = FV_n\left(\frac{1}{1+k}\right)^n$$

$$PV = \$100\left(\frac{1}{1.10}\right)^3$$

$$= \$100(0.7513) = \$75.13$$

This is the numerical solution to solve for PV. 19

Financial Calculator Solution

INPUTS	3	10	?	0	100
	N	I/YR	PV	PMT	FV
OUTPUT			-75.13		

Either PV or FV must be negative. Here PV = -75.13. Put in \$75.13 today, take out \$100 after 3 years.

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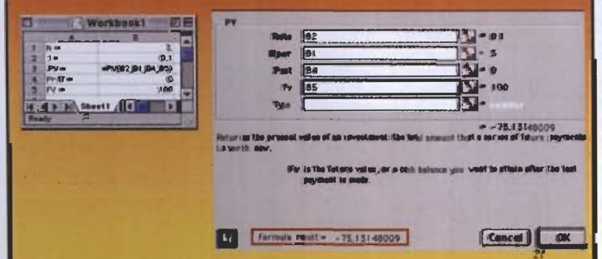
Spreadsheet Solution



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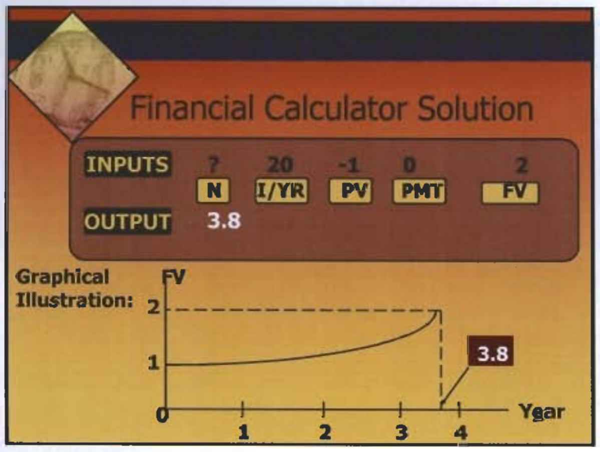
If sales grow at 20% per year, how long before sales double?

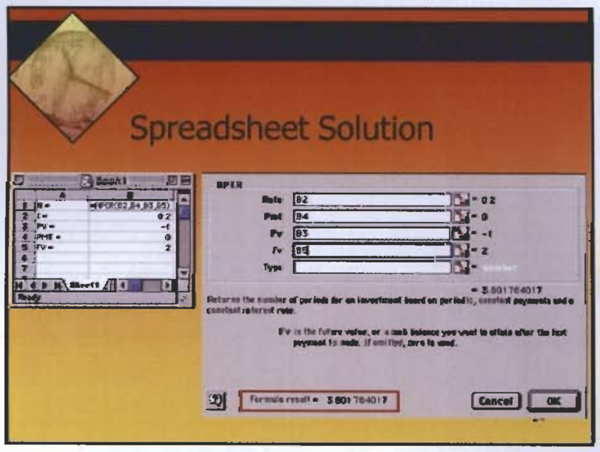
Solve for n:

$$FV_n = 1(1 + k)^n$$

$$2 = 1(1.20)^n$$

The numerical solution is somewhat difficult.

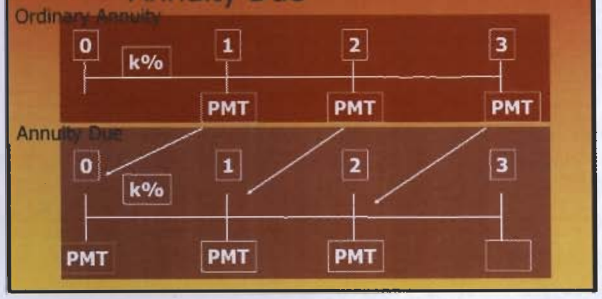




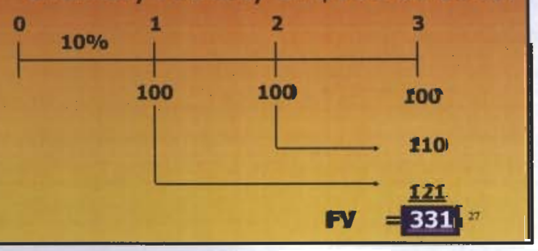
Future Value of an Annuity

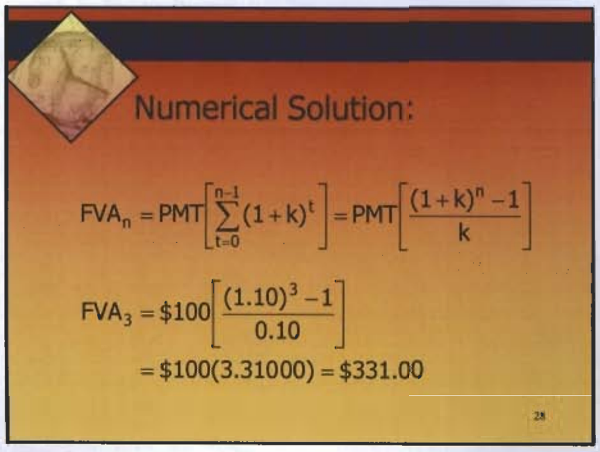
- **Annuity:** A series of payments of equal amounts at fixed intervals for a specified number of periods.
- **Ordinary (deferred) Annuity:** An annuity whose payments occur at the end of each period.
- **Annuity Due:** An annuity whose payments occur at the beginning of each period.

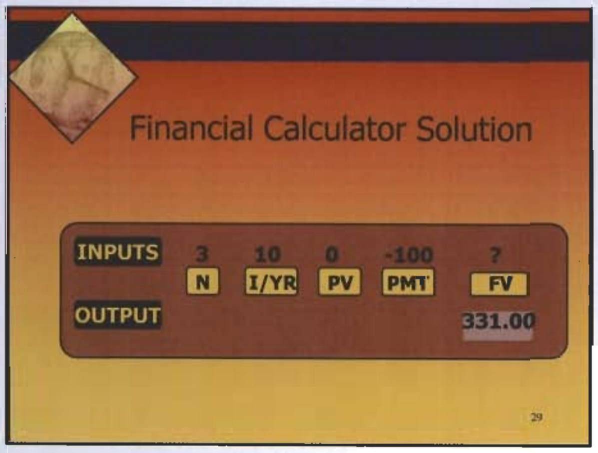
Ordinary Annuity Versus Annuity Due

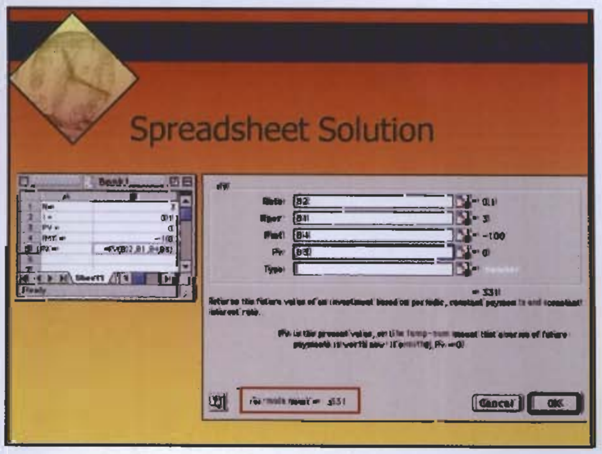


What's the FV of a 3-year Ordinary Annuity of $100 at 10%?




Numerical Solution:
$FVA_n = PMT\left[\sum_{t=0}^{n-1}(1+k)^t\right] = PMT\left[\frac{(1+k)^n - 1}{k}\right]$
$FVA_3 = \$100\left[\frac{(1.10)^3 - 1}{0.10}\right]$
$= \$100(3.31000) = \$331.00$
28

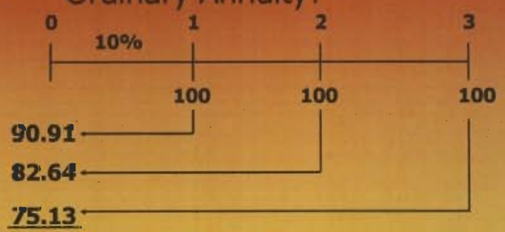

Financial Calculator Solution
INPUTS
3
10
0
-100
?
N
I/YR
PV
PMT
FV
OUTPUT
331.00
29


Spreadsheet Solution

Present Value of an Annuity

- PVA_n = the present value of an annuity with n payments.
- Each payment is discounted, and the sum of the discounted payments is the present value of the annuity.

What is the PV of this Ordinary Annuity?



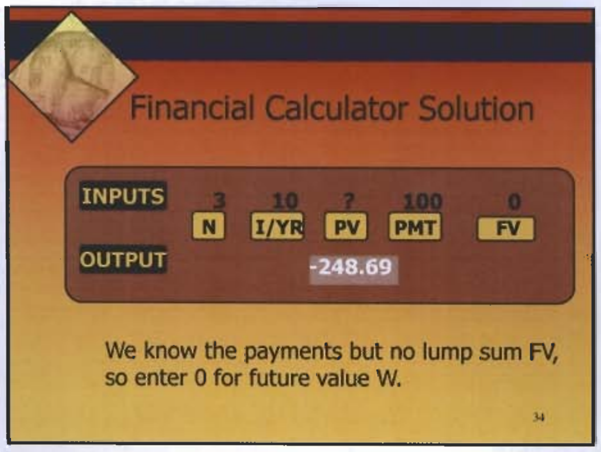
248.69 = PV

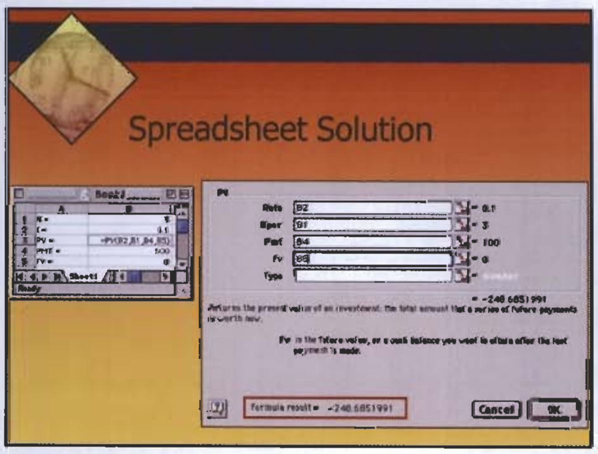
Numerical Solution

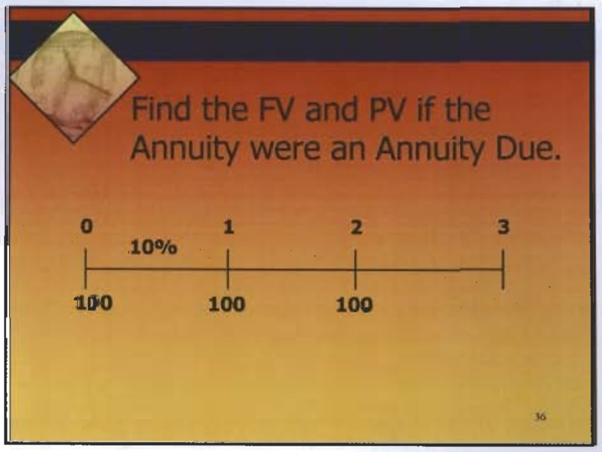
$$PVA_n = PMT\left[\sum_{t=1}^{n}\frac{1}{(1+k)^t}\right] = PMT\left[\frac{1-\frac{1}{(1+k)^n}}{k}\right]$$

$$PVA_3 = \$100\left[\frac{1-\frac{1}{(1.10)^3}}{0.10}\right]$$

$$= \$100(2.48685) = \$248.69$$


Financial Calculator Solution
INPUTS
3
10
?
100
0
N
I/YR
PV
PMT
FV
OUTPUT
-248.69
We know the payments but no lump sum FV, so enter 0 for future value W.
34


Spreadsheet Solution
Cancel
OK


Find the FV and PV if the Annuity were an Annuity Due.
0
1
2
3
10%
100
100
100
36

Numerical Solution

$$PVA_n = PMT\left[\sum_{t=0}^{n-1}\frac{1}{(1+k)^t}\right] = PMT\left\{\left[\frac{1-\frac{1}{(1+k)^n}}{k}\right]\times(1+k)\right\}$$

$$PVA_3 = \$100\left\{\left[\frac{1-\frac{1}{(1.10)^3}}{0.10}\right]\times(1.10)\right\}$$

$$= \$100[(2.48685)\times 1.10]$$

$$= \$100(2.73553) = \$273.55$$

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Financial Calculator Solution

Switch from "End" to "Begin".

Then enter variables to find PVA_3 = \$273.55.

	N	I/YR	PV	PMT	FV
INPUTS	3	10	?	100	0
OUTPUT			-273.55		

Then enter PV = 0 and press FV to find FV = \$364.10.

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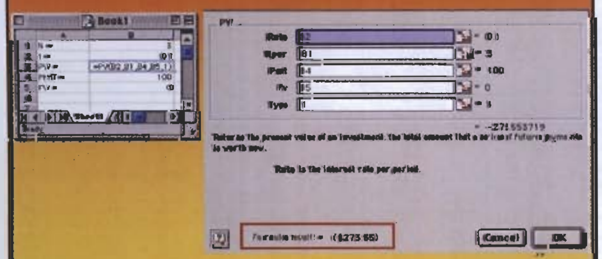
Spreadsheet Solution



Solving for Interest Rates with Annuities

You pay $864.80 for an investment that promises to pay you $250 per year for the next four years, with payments made at the end of each year. What interest rate will you earn on this investment?

0	1	2	3	4
k = ?				
- 864.80	250	250	250	250

40

Numerical Solution

Use trial-and-error by substituting different values of k into the following equation until the right side equals $864.80.

$$\$864.80 = \$250\left[\frac{1 - \frac{1}{(1+k)^4}}{k}\right]$$

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Financial Calculator Solution

INPUTS	4	?	-846.80	250	0
	N	I/YR	PV	PMT	FV
OUTPUT		7.0			

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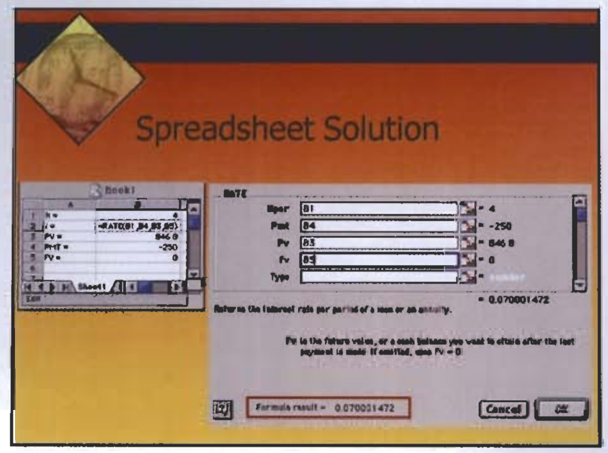
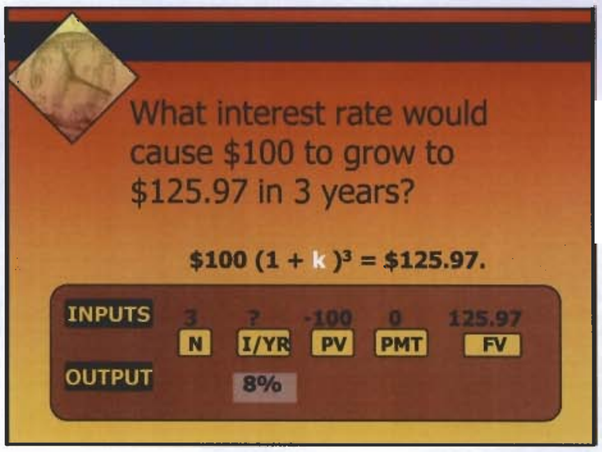
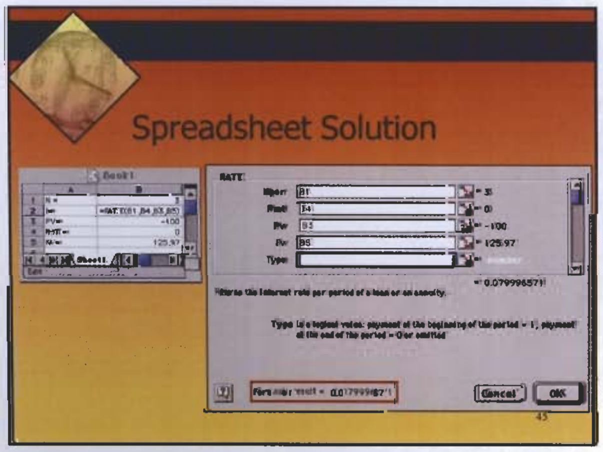

Spreadsheet Solution
Book1
Sheet1
RATE
= 4
= -250
= 0
= 0.070001472
Formula result = 0.070001472
Cancel
OK


What interest rate would cause $100 to grow to $125.97 in 3 years?
$100 (1 + k)³ = $125.97.
INPUTS
3
?
-100
0
125.97
N
I/YR
PV
PMT
FV
OUTPUT
8%


Spreadsheet Solution
Book1
RATE
= 0
= -100
= 125.97
Cancel
OK
45

Uneven Cash Flow Streams

- A series of cash flows in which the amount varies from one period to the next:
  - Payment (PMT) designates constant cash flows—that is, an annuity stream.
  - Cash flow (CF) designates cash flows in general, both constant cash flows and uneven cash flows.

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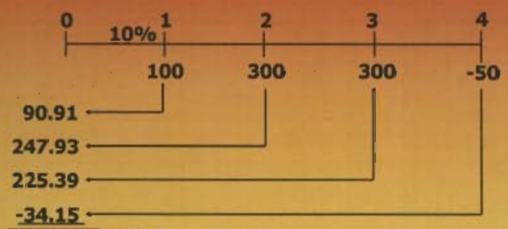
What is the PV of this Uneven Cash Flow Stream?



530.08 = PV

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Numerical Solution

$$PV = CF_1\left[\frac{1}{(1+k)^1}\right] + CF_2\left[\frac{1}{(1+k)^2}\right] + \ldots + CF_n\left[\frac{1}{(1+k)^n}\right]$$

$$PV = 100\left[\frac{1}{(1.10)^1}\right] + 300\left[\frac{1}{(1.10)^2}\right] + 300\left[\frac{1}{(1.10)^3}\right] + (-50)\left[\frac{1}{(1.10)^4}\right]$$

$$= \$100(0.90909) + \$300(0.82645) + \$300(0.75131) + (-\$50)(0.68301)$$

$$= \$530.09$$

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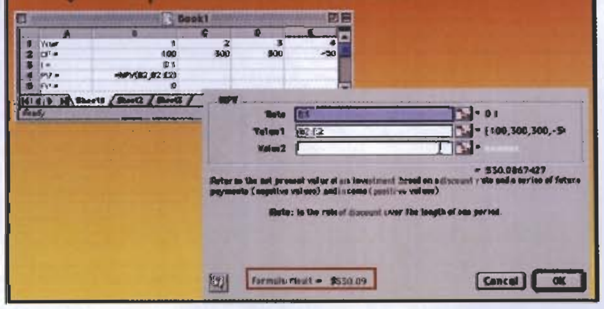
Financial Calculator Solution

- Input in "CF" register:
  - CF0 = 0
  - CF1 = 100
  - CF2 = 300
  - CF3 = 300
  - CF4 = -50
- Enter I = 10%, then press NPV button to get NPV = 530.09. (Here NPV = PV.)

Spreadsheet Solution



Semiannual and Other Compounding Periods

- **Annual compounding** is the process of determining the future value of a cash flow or series of cash flows when interest is added once a year.
- **Semiannual compounding** is the process of determining the future value of a cash flow or series of cash flows when interest is added twice a year.

Will the FV of a lump sum be larger or smaller if we compound more often, holding the stated k constant? Why?

LARGER!

If compounding is more frequent than once a year—for example, semi-annually, quarterly, or daily—interest is earned on interest—that is, compounded—more often.

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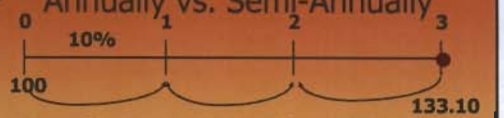
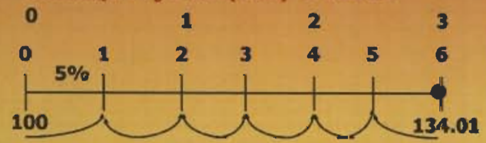
Compounding Annually vs. Semi-Annually



Annually: $FV_3 = 100(1.10)^3 = 133.10$.



Semi-annually: $FV_{6/2} = 100(1.05)^6 = 134.01$.

Distinguishing Between Different Interest Rates

k_{SIMPLE} = Simple (Quoted) Rate
used to compute the interest paid per period

EAR = Effective Annual Rate
the annual rate of interest actually being earned

APR = Annual Percentage Rate = k_{SIMPLE}
periodic rate X the number of periods per year

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How do we find EAR for a simple rate of 10%, compounded semi-annually?

$$EAR = \left(1 + \frac{k_{SIMPLE}}{m}\right)^m - 1$$

$$= \left(1 + \frac{0.10}{2}\right)^2 - 1.0$$

$$= (1.05)^2 - 1.0 = 0.1025 = 10.25\%$$

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FV of $100 after 3 years if interest is 10% compounded semi-annual? Quarterly?

$$FV_n = PV\left(1 + \frac{k_{SIMPLE}}{m}\right)^{m \times n}$$

$$FV_{3\times2} = \$100\left(1 + \frac{0.10}{2}\right)^{2\times3} = \$100(1.34010) = \$134.01$$

$$FV_{3\times4} = \$100\left(1 + \frac{0.10}{4}\right)^{4\times3} = \$100(1.34489) = \$134.49$$

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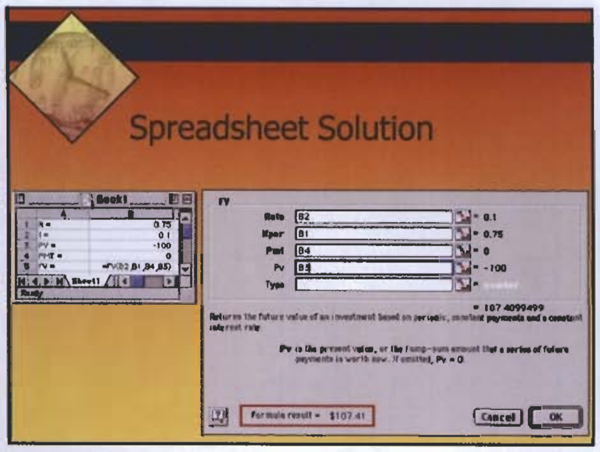
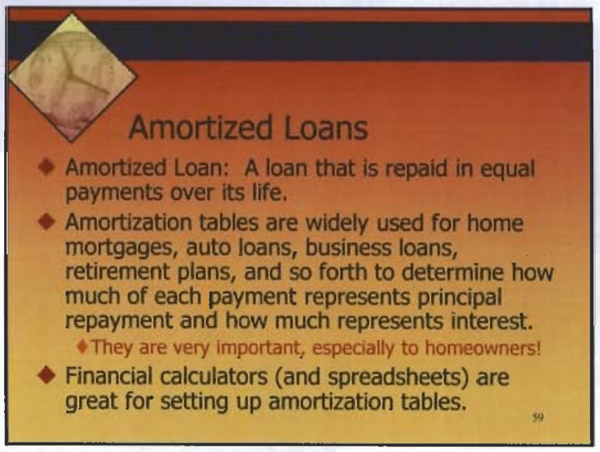
Fractional Time Periods

Example: $100 deposited in a bank at EAR = 10% for 0.75 of the year

0	0.25	0.50	0.75	1.00
10%				
- 100			FV = ?	

INPUTS	0.75	10	-100	0	?
	N	I/YR	PV	PMT	FV
OUTPUT					107.41


Spreadsheet Solution
Book1
0.75
0.1
-100
0
FV
Rate
B2
= 0.1
B1
= 0.75
B4
= 0
Pv
B5
= -100
Type
= 107.4099499
Formula result = $107.41
Cancel
OK


Amortized Loans
Amortized Loan: A loan that is repaid in equal payments over its life.
Amortization tables are widely used for home mortgages, auto loans, business loans, retirement plans, and so forth to determine how much of each payment represents principal repayment and how much represents interest.
They are very important, especially to homeowners!
Financial calculators (and spreadsheets) are great for setting up amortization tables.
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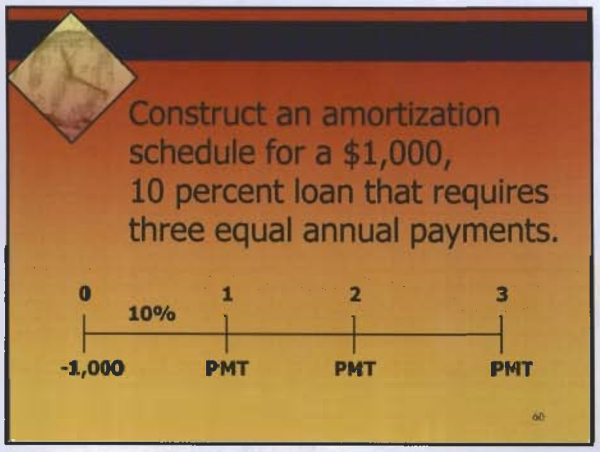
Construct an amortization schedule for a $1,000, 10 percent loan that requires three equal annual payments.
0
1
2
3
10%
-1,000
PMT
PMT
PMT
60

Step 1: Determine the required payments

0	1	2	3
10%			
-1000	PMT	PMT	PMT

INPUTS	3	10	-1000	?	0
	N	I/YR	PV	PMT	FV
OUTPUT				402.11	

Step 2: Find interest charge for Year 1

INT_t = Beginning balance$_t$ (k)

INT_1 = 1,000(0.10) = $100.00

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Step 3: Find repayment of principal in Year 1

Repayment = PMT - INT

= $402.11 - $100.00

= $302.11.

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Step 4: Find ending balance after Year 1

Ending bal. = Beginning bal. - Repayment
= \$1,000 - \$302.11 = \$697.89.

Repeat these steps for the remainder of the payments (Years 2 and 3 in this case) to complete the amortization table.

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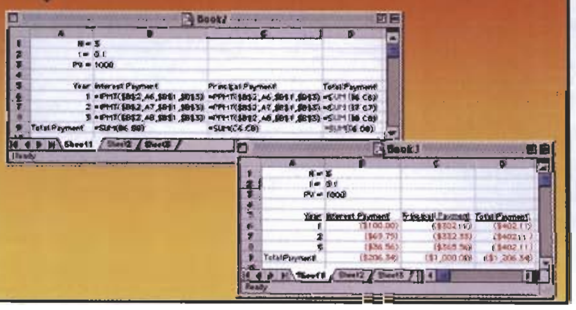
Spreadsheet Solution



Loan Amortization Table 10 Percent Interest Rate

YR	Beg Bal	PMT	INT	Prin PMT	End Bal
1	$1000.00	$402.11	$100.00	$302.11	$697.89
2	697.89	402.11	69.79	332.32	365.57
3	365.57	402.11	36.56	365.55	0.02
Total		1,206.33	206.35	999.98 *	

* Rounding difference

Interest declines, which has tax implications.

Comparison of Different Types of Interest Rates

- k_{SIMPLE}: Written into contracts, quoted by banks and brokers. Not used in calculations or shown on time lines.
- k_{PER}: Used in calculations, shown on time lines. If k_{SIMPLE} has annual compounding, then $k_{PER} = k_{SIMPLE}/1 = k_{SIMPLE}$
- **EAR :** Used to compare returns on investments with different payments per year. (Used for calculations when dealing with annuities where payments don't match interest compounding periods .)

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Simple (Quoted) Rate

- k_{SIMPLE} is stated in contracts. Periods per year (m) must also be given.
- Examples:
  - 8%, compounded quarterly
  - 8%, compounded daily (365 days)

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Periodic Rate

- Periodic rate = $k_{PER} = k_{SIMPLE}/m$, where m is number of compounding periods per year. m = 4 for quarterly, 12 for monthly, and 360 or 365 for daily compounding.
- Examples:
  - 8% quarterly: $k_{PER} = 8/4 = 2\%$
  - 8% daily (365): $k_{PER} = 8/365 = 0.021918\%$

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Effective Annual Rate

Effective Annual Rate:

The annual rate that causes PV to grow to the same FV as under multi-period compounding.

Example: 10%, compounded semiannually:

EAR $= (1 + k_{SIMPLE}/m)^m - 1.0$

$= (1.05)^2 - 1.0 = 0.1025 = 10.25\%$

because $(1.1025)^1 - 1.0 = 0.1025 = 10.25\%$

Any PV would grow to same FV at 10.25% annually or 10% semiannually.

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End of Chapter 3

The Time Value of Money

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