

University of Bahrain
College of Science
Department of Mathematics
Semester 2002/2003
Final Examination

MATHS 102 : Calculus II

Time : 2 hours

Question 1 [6 marks]

- a) Find the limit if it exists

$$\lim_{x \rightarrow \infty} \frac{3^x + 2}{x^3 + 1}$$

- b) Find $\frac{dy}{dx}$ if $\sinh(xy) = \ln(x + y)$

Question 2 [8 marks]

Evaluate the following integrals

a) $\int \sin^2 2x \sin x \, dx$

b) $\int \sqrt{1 + 4x^2} \, dx$

Question 3 [8 marks]

- a) The region in the first quadrant that is enclosed by the x-axis and the curve $y = 3x\sqrt{1-x}$ is revolved about the y-axis to generate a solid. Find the volume of the resulted solid.

(Hint: use the shell method)

- b) Find the length of the curve of $y = \frac{2}{3}(x^2 + 1)^{\frac{3}{2}}$ from $x = 0$ to $x = 2$.

Question 4 [12 marks]

Determine whether the following series is absolutely convergent (A.C.), conditionally convergent (C.C.), or divergent.

a) $\sum_{n=1}^{\infty} \frac{4 \tan^{-1} n}{n^2}$

b) $\sum_{n=2}^{\infty} \left(1 - \frac{1}{n}\right)^n$

c) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{\sqrt{n}}$

Question 5: [8 marks]

Given the function $f(x) = \ln(2 - x)$.

- a) Find a Maclaurin series for $f(x)$.

- b) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} (1-x)^n$.

Question 6: [8 marks]

- a) Find a polar equation that has the same graph as the equation $x^2 = 8y$.

- b) Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$.