

2002/2003
Final Examination
MATHS 104

Question 1

Evaluate

- (a) $\frac{dy}{dx}$ if $y = (2x + 1)^{x+2}$.
- (b) $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x^2 + 4x - 5}$.
- (c) $\frac{dy}{dx}$ if $y = \log_2(8x + 5)^2$.
- (d) $\frac{dy}{dx}$ at the point $(2, 1)$ if $x + xy + y = 5$.

Question 2

(a) If the marginal cost of a firm is given by

$$C'(q) = 0.002q + 2$$

- (i) Find the cost function of the firm if the fixed cost is \$50.
- (ii) Find the cost of producing 1000 units.
- (b) If in addition the firm sells all units it produces at \$8 per unit.
- (iii) Find the revenue and profit as a function of q .
- (iv) Find the production volume q so that the profit is maximum.
- (v) What is the value of the maximum profit?

Question 3

- (a) Find the area between the curve $y = x^2 + 1$ and the line $y = 2x + 9$.
- (b) For the demand relation $q = (1 - p - p^2)$. Find the elasticity of the demand when $p = \frac{1}{4}$ and classify it.
- (c) Find the value of p for which the elasticity $\eta = -1$ in part (b).

Question 4

(a) The supply and demand functions for a certain product are

$$S : p = 20 + 2.5q$$

$$D : p = \frac{280}{q + 2}$$

Determine the consumer's and producers' surplus, assuming the market equilibrium has been established.

- (b) The demand function of a firm's product is $p = 45 - 0.12q$. The cost of producing q units is given by $C(q) = 300 + 5q$. Find the average profit over the sales interval $q = 100$ to $q = 200$.

Question 5 [10 marks]

Evaluate the following:

(a)
$$\int \left(e^{2x+1} + \frac{1}{x-1} + (2x+3)^3 \right) dx.$$

(b)
$$\int x^2 e^{3 \ln x} dx.$$

(c)
$$\frac{d}{dx} \left(\int_1^2 x^2 \ln \sqrt{x^2 + 1} \right).$$

(d)
$$\int (x+1)e^x dx.$$

(e)
$$\frac{d}{dx} \left(\int_x^2 e^{t^2} dt \right).$$