2002/2003

Final Examination

MATHS 205

Question 1

a) Solve the following differential equation $(x^2 + xy^2)y' - 3xy + 2y^3 = 0$

(Hint: The integrating factor is of the form $\mu = x^m y^n$)

b) Verify that one solution of $xy'' - (2x+1)y' + (x+1)y = 0$ is given by $y_1 = e^x$, and find the general solution.

Question 2

a) If $y = x^r, (x > 0)$ defines solution for the D.E. $x^3 y^{(4)} + 8x^2 y''' + 8xy'' - 8y' = 0$
Find the four linearly independent solutions and write the general solution.

b) Using Laplace transform solve the initial value problem

$$y'' + 2ty' - 4y = 1 \,,\; y(0) = y'(0) = 0$$

Question 3

Consider the equation $3xy'' + (2-x)y' - y = 0$, $x > 0$

a) Show that $x = 0$ is regular singular point and find the roots of the indicial equation.

b) Using frobenius method find one solution corresponding to the larger root of the indicial equation in part (a).

Question 4

Consider the n^{th} order D.E. $y^{(n)} - y' - \dfrac{(n-1)}{x} y = 0$ $\quad (x > 0, n \geq 2)$ **(1)**

a) By setting $y = vx$ and $v^{(n-1)} - v = w$ **(2)**
obtain a first-order D.E. Satisfies by w.

b) Solve the first-order D.E. which satisfies by w in part **(a)** subject to the initial condition $w(1) = 0$

c) Use the result of part **(b)** to solve equation **(2)** and consequently obtain the set $\{y_1, y_2, ..., y_{n-1}\}$ of solutions for equation **(1).**