University of Bahrain
College of Science
Mathematics department
Second Semester 2004-2005

Final Examination

Math 211
Duration: 2 hours
Date: 15th Jun, 2005
Max. Mark: 50

Name:	**I.D.No:**	**Section:**

Marking Scheme

Questions	Max. Mark	Mark. Obtained
1	12	
2	12	
3	12	
4	14	
Total	50	

Question 1: **[12 marks]**

a) Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ 1 & 1 & a^2 - 5 \end{bmatrix}$. Determine when A^{-1} exists.

b) Prove that $H = \{\, p \in P_2 \;:\; p(1) = 2\,p(0) \,\}$ is a subspace of P_2.

c) Let $H = \text{Span}\{(1,1,0)\,,\,(1,0,1)\}$. Find m if the vector $v = (m, -(1+m), -m)$ belongs to H.

Question 2: **[12 marks]**

In the vector space $V = \mathrm{P}_2$, consider $B = \{1 , X\}$ and $B' = \{p = 2 - X \;,\; q = 1 - 2X \}$

a) Prove that B' is a basis of V.

b) Find the transition matrix from B' to B.

c) Find the transition matrix from B to B'.

d) Find the coordinates of $2 + X$ with respect to the basis B'.

Question 3: **[12 marks]**

Let $T : \mathrm{IR}^2 \rightarrow \mathrm{IR}^3$ be the function defined by $T(x , y) = (2x - y , 4x , y)$.

a) Show that T is a linear transformation.

b) Find the kernel of T.

c) Find the range of T.

<u>Question 4</u>: [14 marks]

Consider the following matrix

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$$

a) Find the eigenvalues of A.

b) Find a basis for each eigenspace of A.

c) Is there an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix? Explain.