University of Bahrain
College of Science
Department of Mathematics
First Semester 2010-2011
MATH 311
(Group Theory)
Test 1
Sunday 7 November 2010
Instructor: Dr Khalid Amin
13:00-14:00

Name	_____
Id Number	

Instructions

1. Please write your name and your university identity number in the space provided above.

2. Make sure that your copy of this test consists of 6 pages and 5 different questions.

3. In Question 1, you first mark your answers by **T(True)** or **F (False)** and then justify your claims.

In Questions 4-5, you must show the details of your solutions to the problems.

	Maximum Points Possible	You Scored
Question 1	05	
Question 2	04	
Question 3	05	
Question 4	07	
Question 5	04	
Total	25	

Good Luck

Question 1 [05 points]

Mark each of the following statements as **T(True)** or **F (False)**.

Briefly explain why.

1. The empty set can be considered a group under any binary

operation._____.

2. Any group has at most two subgroups._____.

3. $\mathbb{N}^+=\{1,2,3,\dots\}$ is a group under the *ordinary* addition._____.

4. Any group with *exactly* two generators must be infinite._____.

5. $\mathbb{Q}$ is a cyclic group ._____.

Question 2 [04 points]

In the following, give an example or say no such a thing exists.

1. An infinite abelian group.
2. A finite non-abelian group.
3. A cyclic group of order 2010.
4. A non-abelian group of order 24.

Question 3 [05 points]

Construct the Cayley Table for a group $G = \{e, a, b\}$ of order 3.

Question 4 [07 points]

Let

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 7 & 8 & 9 & 4 & 5 & 2 & 1 & 6 \end{pmatrix}$$

(a) Write σ as a product of disjoint cycles.

(b) Is σ even or an odd permutation?

(c) Find the order of σ.

(d) Find the inverse of σ.

(e) Compute σ^{-2010}.

Question 5 [04 points]

Let H and K be subgroups of a group G.

(a) Show that in general HK is not a subgroup of G.

(b) Give two conditions under which HK is a subgroup of G.