

University of Bahrain  
College of Science  
Mathematics department  
First Semester 2006-2007

Final Examination

Math 352  
Date: 13 / 01 / 2005

Max. Marks: 50  
Duration: 2 hours

<b>Name:</b>
<b>ID Number:</b>

**Instructions:**

- 1) Please check that this test has 5 questions and 6 pages.
- 2) Write your name, student number, and section in the above box.

Question	Max. Marks	Marks obtained
1	10	
2	10	
3	10	
4	10	
5	10	
<b>Total</b>	<b>50</b>	

**Good Luck**

**Question 1: [5 + 5 marks ]**

**a)** Let  $a$  and  $b$  be two integers such that  $\gcd(a,52) = \gcd(b,52) = 1$ . Prove that  $a^{12} - b^{12}$  is divisible by 52.

**b)** If  $\gcd(a,b) = 2$ , find  $\text{lcm}(a^2b + a, ba^2 + b)$ .

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**Question 2: [5 + 5 marks]**

- a) Find the remainder when  $2^{2^n} + 1$  is divided by 12, for  $n \geq 1$ .
- b) Find an integer having the remainder 2, 3, 4, 5 when dividing by 3, 4, 5, 6 respectively.
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**Question 3: [5 + 5 marks]**

- a) Find all prime numbers  $p$  for which  $7p + 1$  is a perfect cube.
- b) Divide 264 into the sum of two positive integers such that one is divisible by 24 and the other by 9.
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**Question 4: [5 + 5 marks ]**

Let  $A = a(17)^{2n+1} + (27)^{2n+2}$ , where  $a \in \{1, 2, \dots, 9\}$ .

- a) Determine  $a$  so that 5 divides  $A$ .
  - b) Find the units digit of  $A$  when  $a = 4$ .
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**Question 5:** [5 + 5 marks]

a) Prove:  $\left(1 + \frac{1}{n}\right)^n \leq \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \dots + \frac{1}{n!}$ , where  $n \geq 1$  is an integer.

b) Deduce that  $\left(1 + \frac{1}{n}\right)^n \leq 3$  (Hint: use the fact that  $2^{n-1} \leq n!$ ).

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