University of Bahrain
College of Science
Mathematics department
First Semester 2007-2008

Final Examination

Math 352
Duration: 2 hours
Date: 29 / 01 / 2008
Max. Mark: 50

Name:	
ID Number:	**Section:**

Instructions:

1) **Please check that this test has 5 questions and 6 pages.**
2) **Write your name, student number, and section in the above box.**

Marking Scheme

Questions	**Max. Mark**	**Mark. Obtained**
1	**10**	
2	**10**	
3	**10**	
4	**10**	
5	**10**	
Total	**50**	

Good Luck

Question 1: **[5 + 5 marks]**

a) Prove that 8 divides $n^2 - 1$ for every odd integer n.

b) Find the remainder when $2(41!) + 2^{84}$ is divided by 43.

Question 2: **[5 + 5 marks]**

a) For any integer a, find the possible units digit of $a^2 + a + 1$.

b) The Fibonacci numbers a_o, a_1, a_2 … are defined by $a_o = 0$, $a_1 = 1$, and $a_n = a_{n-1} + a_{n-2}$

Prove, by induction, that $a_n \geq \alpha^{n-2}$ for $n \geq 1$, where $\alpha = \frac{1+\sqrt{5}}{2}$. (Hint: $\alpha^2 = \alpha + 1$).

Question 3: **[5 + 5 marks]**

a) Find the least positive integer a so that $6 / (a + 1)$, $5 / (a + 2)$ and $11 / (a + 7)$.

b) By using linear congruences, solve the congruence equation: $4x^2 \equiv 1 \pmod{11}$.

Question 4: **[5 + 5 marks]**

a) If gcd(a , b) = 3 , show that gcd(a^{n+1}, b^n) = 3^n or gcd(a^{n+1}, b^n) = 3^{n+1}.

b) Let $n = 2p$ for some prime number $p > 2$. Show that if a is a positive integer such that gcd(n , a) = 1, then $a^{n-1} \equiv a \pmod{n}$.

Question 5: [5 + 5 marks]

a) Let a and b be two positive integers. Prove that if a / b, then $(2^a - 1) / (2^b - 1)$, and deduce that, if $2^m - 1$ is prime, then m is prime.

b) Prove that if $n > 2$ is an integer such that $(n-1)! \equiv -1 \pmod{n}$, then n is prime.