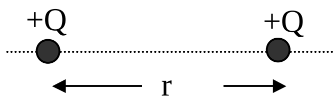
University of Bahrain	Physics 102 Final Exam Key	Fall 2004
Department of Physics	13/1/2005	8:30 - 10:30

$e = 1.6\times10^{-19}$C,	$m_e = 9.11\times10^{-31}$Kg,	$m_p = 1.67\times10^{-27}$Kg
$k = 9\times10^{9}$ Nm^2/C^2,	$\varepsilon_0 = 8.84\times10^{-12}$ C^2/Nm^2,	$\mu_0 = 4\pi\times10^{-7}$ T.m/A

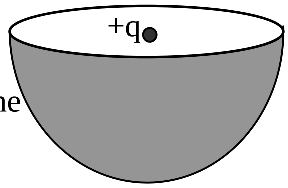
Part I: 10 MCQ (5 marks each)

Q1- Two identical charges are placed at a distance r = 0.3 m as shown in the figure. If the magnitude of the electric force between them is 4 N, then the value of charge in (μC) is:



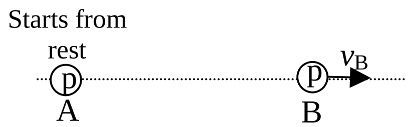
(A) 6.32 (B) 9.49 (C) 12.65 (D) 15.81 (E) 18.97

Q2- A positive point charge +q is placed at the centre of inverted hemispherical surface (half spherical shell) as shown in the figure. The electric flux through the hemispherical surface is:



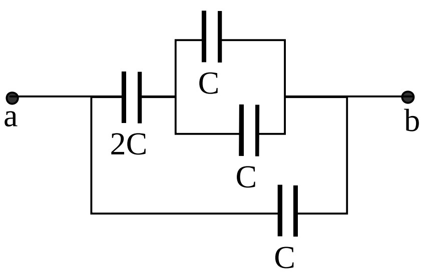
(A) Zero (B) q/ε_0 (C) $q/2\varepsilon_0$ (D) $q/3\varepsilon_0$ (E) $q/4\varepsilon_0$

Q3- A proton is accelerated from rest at point A. If the charge reaches a speed of $v_B = 1\times10^{5}$ m/s at point B, then the electric potential difference ΔV in (V) between points A and B is:



(A) -104.4 (B) -208.8 (C) -313.1 (D) -417.5 (E) -521.9

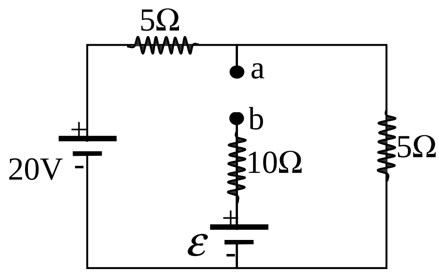
Q4- In the circuit shown, if C= 8 μF then the equivalent capacitance (in μF) between the two points a and b is;



(A) 4 (B) 8 (C) 12 (D) 16 (E) 20

Q5- A copper wire curries a current of 3 A at 20°C. If the temperature coefficient of copper is $\alpha = 0.004°C^{-1}$, then the current (in A) in the wire when its temperature increases to 100 °C is: (Assume that the voltage supplied to the wire remains the same)

(A) 0.76 (B) 1.52 (C) 2.27 (D) 3.03 (E) 3.79



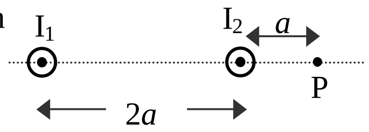
Q6- In the circuit shown, if ε = 12 V, then the value of potential difference (in V) between the two points a and b is:

(A) 2 (B) 4 (C) 6 (D) 8 (E) 10

Q7- A charge q = 2 μC is moving with a velocity $\vec{v} = [3\hat{i} + 4\hat{j}]\times 10^6$ m/s. If the charge enters a magnetic field $\vec{B} = 0.4\hat{k}$ T, then the magnitude of magnetic force (in N) on the charge is:

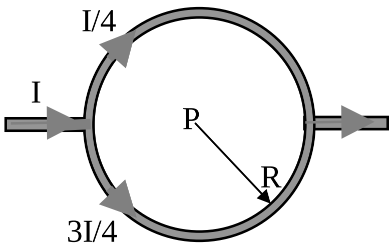
(A) 4 (B) 8 (C) 12 (D) 16 (E) 20

Q8- Two long, straight, parallel wires carries currents I_1=3I and I_2=I, both directed out of the page as shown in the figure. If the wires are separated by a distance 2*a* then the magnitude of the net magnetic field at point P that is located at a distance *a* from I_2 is given by;



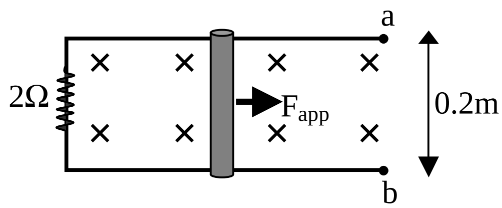
(A) Zero (B) $\mu_0 I/2a$ (C) $\mu_0 I/a$ (D) $\mu_0 I/4a$ (E) $3\mu_0 I/a$

Q9- A current I enters a ring of radius R = 0.2m. The current splits into unequal portions I_1=I/4 and I_2=3I/4 as shown in the figure. If I= 5 A, then the magnitude of the net magnetic field (in μT) at the center of the ring (at point P) is:



(A) 3.93 (B) 7.85 (C) 11.78 (D) 15.7 (E) 19.63

Q10- A rod (length 0.2 m) moves on two frictionless horizontal conducting rails. A 0.5 T magnetic field is directed into the plane of the loop as shown in the figure. If the rod is moving at a constant speed when F_{app}=1 N, then the value of induced voltage (in V) between points a and b is:



(A) 20 (B) 40 (C) 60 (D) 80 (E) 100

Solution of the above questions

Q1: $F = k\,Q^2 / r^2$, $Q = 6.32\mu c$

Q2: $\phi = \dfrac{q}{\in_o}(sphere), \phi = \dfrac{1}{2}\dfrac{q}{\in_o}(hemisphere)$

Q3: Conservation of energy: $T_A + U_A = T_B + U_B$, $Zero + eV_A = \dfrac{1}{2}mv_B^2 + eV_B \;\; \therefore V_B - V_A = -52.2V$

Q4: $C_{eq} = \left\{C + \dfrac{(2C)(2C)}{2C+2C}\right\} = 2C = 16\mu F$

Q5: $\left.\begin{aligned} I_{100} &= \dfrac{V}{R_{100}} \\ I_{20} &= \dfrac{V}{R_{20}} \end{aligned}\right\} I_{100} = \dfrac{I_{20}R_{20}}{R_{20}\left[1+\alpha(80)\right]} = \dfrac{I_{20}}{1+80\alpha} = 2.27A$

Q6: $I = \dfrac{20}{5+5} = 2A$, $V_{ab} = \sum I_k R_k - \Sigma_k \varepsilon_k$, $V_{ab} = 2(5) - (12) = -2V$, $|V_{ab}| = 2V$

Q7: $\vec{F}_m = q\left(\vec{\upsilon} \wedge \vec{B}\right) = 2\times 10^{-6}\left[\left(3\vec{i} + 4\vec{j}\right)10^6 \wedge 0.4\vec{k}\right] = 0.8\left(4\vec{i} - 3\vec{j}\right), \qquad F_m = 4N$

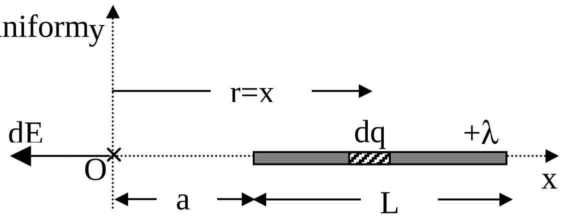
Q8: $B_p = B_1 + B_2 = \dfrac{\mu_o I_1}{2\pi(3a)} + \dfrac{\mu_o I_2}{2\pi(a)} = \dfrac{\mu_o I}{\pi a} Upwards$

Q9: $B_{arc} = \dfrac{\mu_o I}{4\pi R}\theta, B = B_2 - B_1 = \dfrac{\mu_o(3I/4)}{4\pi R}\pi - \dfrac{\mu_o(I/4)}{4\pi R}\pi, B = 3.93\mu T \odot$

Q10: $F_{app} = F_{mag} = I_i LB$ and $\varepsilon_i = I_i R$, $\varepsilon_i = (R/LB)F_{app} = 20V$

Part II: Problem 1	(10 marks)

A rod of length L = 0.5 m lies along the x-axis and has a uniform charge density λ=100 nC/m. Calculate:



(a) The total charge on the rod.

Q=λ L =100*0.5 =50 nC

(b) The electric field (**give its magnitude and direction)** at the origin if a = 0.1m.

dE= k dq/r^2, dq=λdl=λdx,

$$E = \int \frac{kdq}{r^2} = \int_a^{a+L} \frac{k\lambda dx}{x^2} = k\lambda \int_a^{a+L} \frac{dx}{x^2} = -k\lambda \left.\frac{1}{x}\right|_a^{a+L} = k\lambda \frac{L}{a(a+L)}$$

$$E = \frac{kQ}{a(a+L)} = \frac{9\times10^9 * 50\times10^{-9}}{0.1(0.1+0.5)} = 7500 \quad N/C$$

$$\vec{E} = 7500\left(-\vec{i}\right) N/C$$

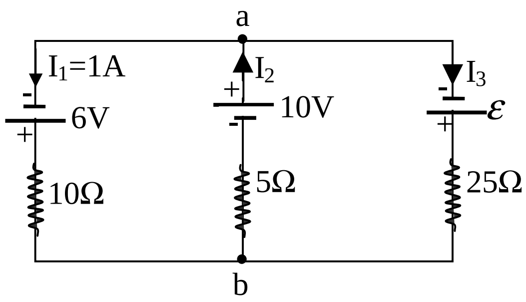
(c) The electric force (**give its magnitude and direction)** on an electron placed at the origin.

$$\vec{F}_e = q\vec{E} = (-e)(7500)\left(-\vec{i}\right) = 1.2\times10^{-15}\,\vec{i}\,N$$

(d) The magnitude of the electron’s initial acceleration at the origin.

$$F = ma, \qquad a = \frac{1.2\times10^{-15}}{9.1\times10^{-31}} = 1.32\times10^{15}\, m/s^2$$

Part II: Problem 2 (15 marks)



In the circuit shown, the current I_1=1 A. Find:

(a) The magnitude of the potential difference between points a and b.

$$V_{ab} = 1\times 10 - 6 = 4V$$

(b) The unknown emf ε.

$$V_{ab} = 4 = -5I_2 + 10, \quad I_2 = 1.2A \quad \therefore I_3 = 0.2$$

$$V_{ab} = 4 = (0.2)(25) - \varepsilon \quad \therefore \varepsilon = 1V$$

$$or, \quad Left\ loop: (1)(10) + 5(I_2) = 16 \quad gives\ I_2 = 1.2\ A$$

$$I_2 = I_1 + I_3 \quad gives\ I_3 = 0.2\ A$$

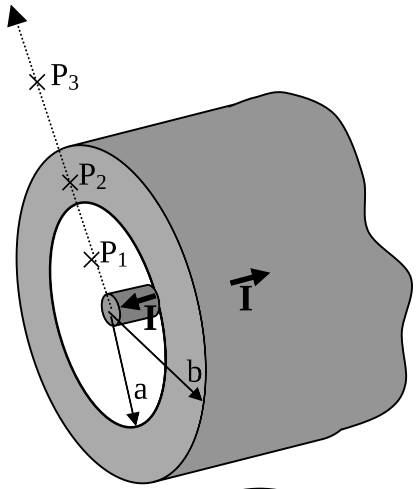
$$Right\ loop: 25I_3 + 5I_2 = 10 + \varepsilon, \quad gives\ \varepsilon = 1V$$

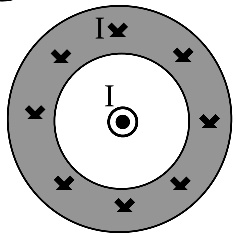
(c) The power dissipated in the 5Ω resistor.

$$P = I_2^2(5) = 7.2\ W$$

Part II : Problem 3 (10 marks)

A coaxial cable consists of a thin conducting wire concentric with a conducting tube with inner radius a = 0.2 m and outer radius b = 0.3 m as shown in the figure. The wire and the tube carry **equal currents I=10A in opposite directions**. The current in the tube is distributed uniformly over its cross section. Calculate the magnitude of the magnetic field at the following points:





Front view

(a) Point P_1 that is located at a distance 0.1 m from the axis of the conductor.

$$\oint B.dl = \mu_o I_{en}$$

$$B \times 2\pi r = \mu_o I, \; B = \frac{\mu_o I}{2\pi r}$$

$$B(P_1) = \frac{4\pi \times 10^{-7} \times 10}{2(\pi)(0.1)} = 20\mu T$$

(b) Point P_2 that is located at a distance 0.25 m from the axis of the conductor.

$$\oint B.dl = \mu_o I_{enc} = \mu_o \left[I - JA_{in}\right]$$

$$B \times 2\pi r = \mu_o \left[I - J\left[\pi\left(r^2 - a^2\right)\right]\right] \qquad J = \frac{I}{A} = \frac{I}{\pi\left[b^2 - a^2\right]}$$

$$B = \frac{\mu_o}{2\pi r}\left[I - \frac{I}{\pi\left[b^2 - a^2\right]}\pi\left(r^2 - a^2\right)\right]$$

$$B = \frac{\mu_o I}{2\pi r}\left[1 - \frac{\left(r^2 - a^2\right)}{\left(b^2 - a^2\right)}\right]$$

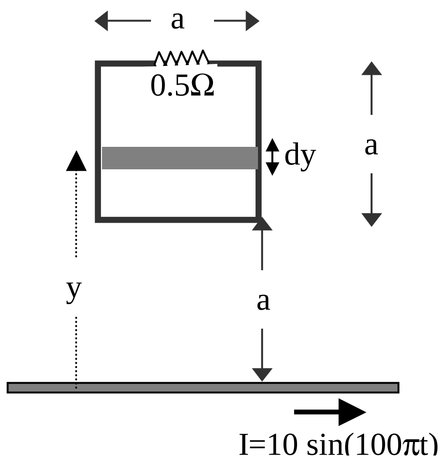
$$B(P_2) = \frac{4\pi \times 10^{-7} \times 10}{2\pi(0.25)}\left[1 - \frac{\left[(0.25)^2 - (0.2)^2\right]}{\left[(0.3)^2 - (0.2)^2\right]}\right] = 4.4\mu T$$

(c) Point P_3 that is located at a distance 0.5 m from the axis of the conductor.

$$\oint B.dl = \mu_o I_{en} = \mu_o \left[I - I\right] = Zero$$

$$B(P_3) = Zero$$

Part II: Problem 4 (15 marks)



In the figure shown, a square loop of side a=0.2m and resistance R=0.5Ω. The loop lies at a distance a above a long, straight wire carrying current that is varying with time according to the relation: I = 10 sin(100πt) where I is in (A) and t in (s).

(a) Show that the magnetic flux (Φ) through the square loop is given by:

$$\Phi = \frac{\mu_0 I}{2\pi} a \ln(2)$$

$$\Phi = \int_s B dA \cos\theta = \int_a^{2a} \frac{\mu_o I}{2\pi y}(a dy)\cos(0)$$

$$\Phi = \int_a^{2a} \frac{\mu_o I a}{2\pi}\frac{dy}{y} = \frac{\mu_o I a}{2\pi} In(2)$$

(b) Determine the induced emf (in μV), as a function of time, in the loop.

$$\varepsilon_{ind} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\left[\frac{\mu_o 2\sin(100\pi t)}{2\pi} Ln2\right] = \left(-4\pi\times10^{-7}\right)(100Ln2)\cos(100\pi t)$$

$$\varepsilon_{ind} = -87.1\cos(100\pi t) \qquad \mu V$$

(c) Determine the maximum induced current (in μA) in the loop.

$$I_{ind} = \frac{\varepsilon_{ind}}{R} = -\frac{87.1}{0.5}\cos(100\pi t) = -174.2\cos(100\pi t)\ \mu A$$

$$[I_{ind}]_{max} = 174.2\ \mu A$$