

Name: \_\_\_\_\_

ID no.: \_\_\_\_\_

Instructor Name: \_\_\_\_\_

Section: \_\_\_\_\_

$$k = 9 \times 10^9 \text{ Nm}^2/\text{C}^2,$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{Nm}^2$$

$$e = 1.6 \times 10^{-19} \text{ C},$$

$$m_e = 9.11 \times 10^{-31} \text{ Kg},$$

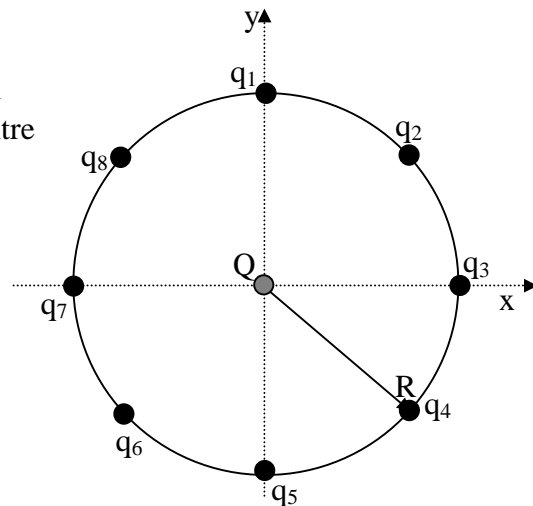
$$m_p = 1.67 \times 10^{-27} \text{ Kg},$$

$$g = 10 \text{ m/s}^2$$

**Problem 1:**

Eight identical charges each of  $q = +10 \mu\text{C}$  are located on the rim of a circle ( $R = 0.3\text{m}$ ) while a charge  $Q = 5 \mu\text{C}$  is located at the centre as shown in the figure.

- (a)- Show on the diagram all the electric forces that are acting on the charge  $Q$ .
- (b)- Find the net electric force exerted on the charge  $Q$ .



**All eight forces are equal in magnitude, and**

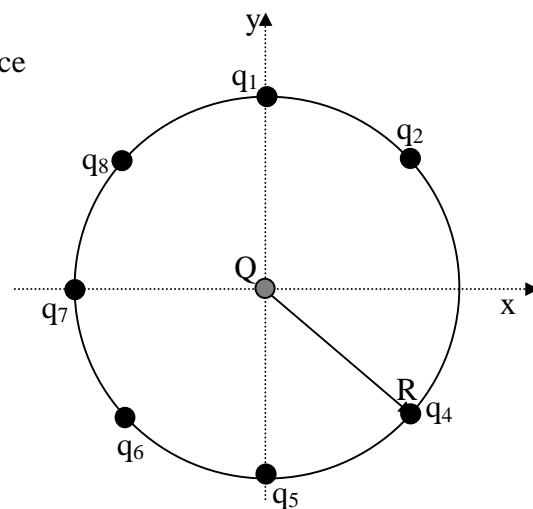
$$\vec{F}_{net} = \sum_{k=1}^8 \vec{F}_k = \text{Zero}$$

- (c)- If the charge  $q_3$  is removed, then calculate the net electric force exerted on the charge  $Q$  (**give its magnitude and direction**).

$$\vec{F}_{net} = \sum_{k=1}^8 \vec{F}_k = \vec{F}_7$$

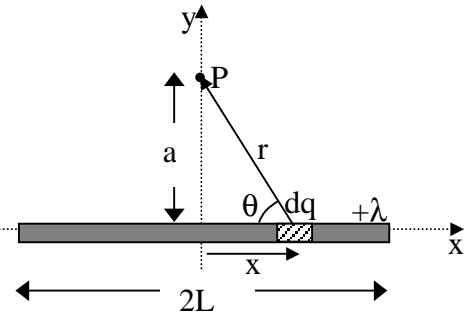
$$F_7 = \frac{kQq_7}{R^2} = 5\text{N}$$

$$\vec{F}_{net} = F_7 \hat{i} = 5\hat{i} \text{ (N)}$$



**Problem 2:**

(I)- A positive charge Q is uniformly distributed on a thin rod of length 2L. If the rod is placed as shown in the figure then find the **magnitude and direction** of the electric field at point P that is located on the bisector line.



$E_x = \text{Zero, By Symmetry}$

$$dE_y = dE \sin \theta$$

$$E_y = \int \frac{a \, kdq}{r^2}$$

$$r = \sqrt{x^2 + a^2} \quad dq = \lambda dx$$

$$E_y = k\lambda a \int_{-L}^L \frac{dx}{(x^2 + a^2)^{3/2}} = k\lambda a \left[ \frac{x}{a^2 \sqrt{x^2 + a^2}} \right]_{-L}^L$$

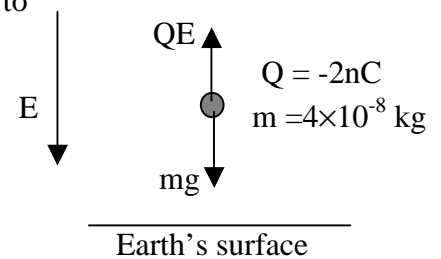
$$E_y = \frac{kQ}{a\sqrt{L^2 + a^2}}, \quad Q = 2\lambda L$$

$$\vec{E}_p = \frac{kQ}{a\sqrt{L^2 + a^2}} \hat{j}$$

You may use the following:

$$\int \frac{du}{(u^2 + c^2)^{3/2}} = \frac{u}{c^2 \sqrt{u^2 + c^2}}$$

(II)- Find the **magnitude and direction** of the electric field required to suspend (remain stationary) a charge  $Q = -2 \text{ nC}$  and mass  $m = 4 \times 10^{-8} \text{ kg}$  in air near the earth's surface.



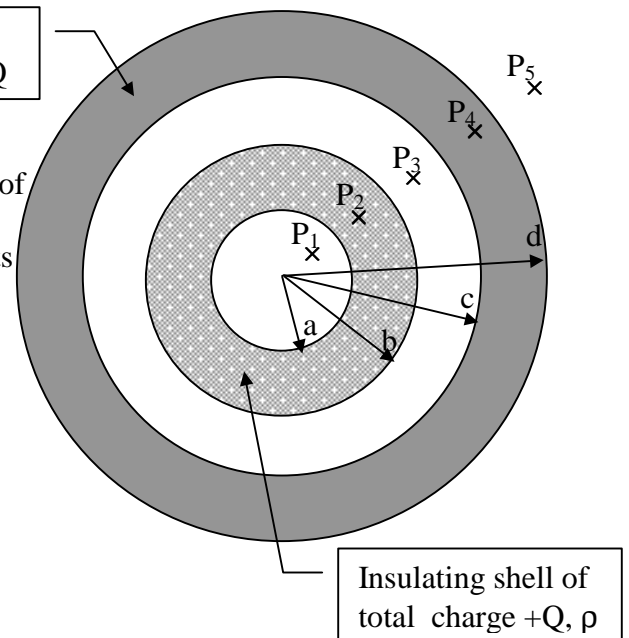
E must be downward so  $\vec{F}_e$  is upward

$$F_e = W, QE = mg$$

$$\therefore E = 200 \frac{N}{C}$$

**Problem 3:**

A thick insulating spherical shell of radii  $a$  and  $b$  is concentric with another thick conducting spherical shell of radii  $c$  and  $d$  as shown in the figure. The insulating shell has a total charge  $+Q$  distributed uniformly throughout its volume (i.e. has a uniform volume charge density  $\rho$ ). The conducting shell has a net charge of  $+3Q$ .



(a)-Calculate the electric field at the following points:

**Point P<sub>1</sub>:**

$$\oint \vec{E}_1 \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} = \text{zero}$$

$\therefore E_1 = \text{Zero}$ , No charges inside

**Point P<sub>2</sub>:**

$$\oint \vec{E}_2 \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}, \quad E_2 \times 4\pi r^2 = \frac{\rho V_{in}}{\epsilon_0} = \rho \frac{4\pi}{3} (r^3 - a^3)$$

$$E_2 = \frac{\rho}{3\epsilon_0} \frac{(r^3 - a^3)}{r^2}$$

**Point P<sub>3</sub>:**

$$\oint \vec{E}_3 \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}, \quad E_3 \times 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E_3 = \frac{kQ}{r^2}$$

**Point P<sub>4</sub>:**

$$\oint \vec{E}_4 \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}, \quad E_4 \times 4\pi r^2 = \frac{Q - Q}{\epsilon_0} = \text{Zero}$$

$E_4 = \text{Zero}$ , inside a conductor

**Point P<sub>5</sub>:**

$$\oint \vec{E}_5 \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}, \quad E_5 \times 4\pi r^2 = \frac{+Q - Q + Q + 3Q}{\epsilon_0} = \frac{4Q}{\epsilon_0}$$

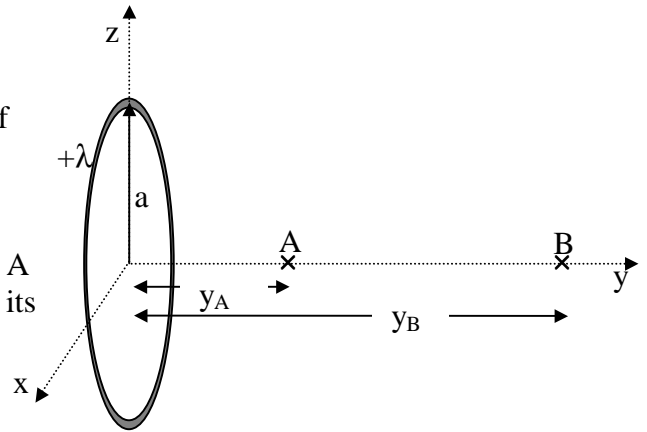
$$E_5 = \frac{4kQ}{r^2}$$

(b) Calculate the surface charge densities on the inner ( $\sigma_c$ ) and outer ( $\sigma_d$ ) surfaces of the thick conducting shell.

$$\sigma_c = \frac{Q_c}{A} = \frac{-Q}{4\pi c^2}, \quad \sigma_d = \frac{Q_d}{A} = \frac{Q + 3Q}{4\pi d^2} = \frac{4Q}{4\pi d^2}$$

**Problem 4:**

A 10 nC charge is uniformly distributed on a thin ring of radius  $a = 0.3$  m. The ring is placed in the x-z plane as shown in the figure.



- (a)- Find an expression for the electric potential at point A that is located on the ring axis at a distance  $y_A$  from its centre.

$$V_A = \int \frac{k dq}{r} = \int \frac{k \lambda d\ell}{\sqrt{a^2 + y_A^2}}$$

$$V_A = \frac{k \lambda}{\sqrt{a^2 + y_A^2}} \int d\ell = \frac{k \lambda L}{\sqrt{a^2 + y_A^2}} = \frac{k Q}{\sqrt{a^2 + y_A^2}}$$

- (b)- Calculate the electric potential difference  $V_B - V_A$  between points A and B if  $y_A = 0.3$  m and  $y_B = 0.9$  m.

$$V_A = \frac{k Q}{\sqrt{a^2 + y_A^2}} = 212.13 \text{ V}$$

$$V_B = \frac{k Q}{\sqrt{a^2 + y_B^2}} = 94.86 \text{ V}$$

$$\Delta V = V_B - V_A = 94.86 - 212.13 = -117.3 \text{ V}$$

- (c)- If a charge of  $q = 2 \mu\text{C}$  and mass  $m = 2 \times 10^{-6}$  kg starts from rest at point A, then find its final speed at point B.

$$\Delta K = -\Delta U = -q \Delta V$$

$$\frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = -2 \times 10^{-6} (-117.3), \quad v_A = 0$$

$$V_B = 15.3 \text{ m/s}$$