

Solution of Final Exam 17/06/2004

Part I: MCQ

$$\text{Q1: } F = k \frac{Q^2}{(0.15)^2} = 4 \times 10^{11} Q^2$$

$$F_{\text{net}} = 2F \cos 30^\circ = 6.928 \times 10^{11} Q^2$$
$$F_{\text{net}} = 24.9 \mu\text{N}, \text{ (for } Q = 6 \text{ n C)}$$

$$\text{Q2: } E(2\pi r \ell) = \frac{(-80 - 30) \times 10^{-9} \ell}{8.85 \times 10^{-12}}, \quad E = \frac{1980}{r}$$

$$E = 16.5 \text{ kN/C}, \text{ (for } r = 120 \text{ mm)}$$

$$\text{Q3: } V_A = k \left(\frac{24 \times 10^{-9}}{0.32} - \frac{67 \times 10^{-9}}{0.18} \right) = -2675 \text{ V}$$

$$V_B = k \left(\frac{24 \times 10^{-9}}{0.4} - \frac{67 \times 10^{-9}}{0.9} \right) = -130 \text{ V}$$

$$\Delta V = 2545 \quad \therefore W = q_3 \Delta V = 5.09 \mu\text{J}, \text{ (for } q_3 = 2 \text{ nC)}$$

$$\text{Q4: } Q = CV = \frac{\epsilon_0 A}{d} E d = \epsilon_0 A E = 61.9 \text{ nC}, \text{ (for } E = 10^6 \text{ V/m)}$$

$$\text{Q5: } J = \frac{I}{A} = \frac{V}{RA} = \frac{V}{\left(\frac{\rho_1 \ell_1}{A} + \frac{\rho_2 \ell_2}{A} \right) A} = \frac{V}{(\rho_1 \ell_1 + \rho_2 \ell_2)}$$

$$J = 625 \text{ A/m}^2, \text{ (for } V = 2 \text{ V)}$$

$$\text{Q6: } I = \frac{\epsilon}{5 + \frac{3 \times 6}{3 + 6}} = 0.1428 \epsilon, \quad \epsilon = (0.1428 \epsilon) \times 5 + V$$

$$V = 0.2857 \epsilon \quad \therefore I_{2\Omega} = \frac{V}{3} = 0.0952 \epsilon$$

$$P = I_{2\Omega}^2 \times 2 = 0.16 \text{ W}, \text{ (for } \epsilon = 3 \text{ V)}$$

Q7:

$$I = \frac{12}{10+2} = 1.0A, V_{ab} = +1 \times 10 - (\varepsilon) = 10 - \varepsilon, V_{ab} = 2V, \text{ (for } \varepsilon = 8V \text{)}$$

Q8:

$$qvB = qE \quad \therefore v = \frac{E}{B} = \frac{4 \times 10^3}{B} \text{ m/s}$$

$$K = \frac{1}{2}mv^2 = \frac{\frac{1}{2}(9.1 \times 10^{-31}) \left(\frac{4 \times 10^3}{B} \right)^2}{1.6 \times 10^{-19}} \\ = \frac{4.55 \times 10^{-5}}{B^2} = 0.71 \text{ eV, (for } B = 8mT \text{)}$$

$$B_1 = \frac{\mu_0 \times 20}{2\pi(0.1)} = 4 \times 10^{-5}T, \quad B_2 = \frac{\mu_0 I_2}{2\pi(0.1)} = 0.2 I_2 \times 10^{-5}T$$

Q9: $B = \sqrt{B_1^2 + B_2^2} = 41.2 \mu T, \text{ (for } I_2 = 5A \text{)}$

Q10: $B = \frac{\mu_0 I r}{2\pi R^2} = 2.4 I \times 10^{-5}T = 24 \mu T, \text{ (for } I = 1.0A \text{)}$

$$\Phi = BA = 0.2(4 + 3t^2), \quad E = -N \frac{d\Phi}{dt}$$

Q11: $E = -6t = -6V, \text{ (for } t = 1.0s \text{)}$

Q12:

$$F = F_m = I_i L B \quad \therefore L B = 0.6 / I_i$$

$$I_i = \frac{\varepsilon_i}{12} = \frac{B L v}{12} = \frac{\left(\frac{0.6}{I_i} \right) v}{12} \quad \therefore I_i = \sqrt{0.05v}$$

$$I_i = 0.22A, \quad \text{(for } v = 1.0 \text{ m/s)}$$

Part II:

Problem 1

$$V_1 = \int_R^{3R} k \frac{\lambda dx}{x} = k \lambda \ln 3, \quad V_3 = V_1$$

$$V_2 = \int_0^\pi k \frac{\lambda R d\theta}{R} = k \lambda \pi$$

Problem 2

$$V_{ab} = \frac{9}{12k + 15k} 15k = 5V$$

(a) $Q = CV_c = CV_{ab} = 50 \mu C$

$$\tau = RC = 18k \times 10\mu = 0.18s$$

$$Q = Q_0 e^{-t/\tau}$$

(b) $t = -\tau \ln 0.25 = 0.25 s$

Problem 3

$$B_1 = 0, \quad B_3 = 0$$

$$B_2 = \frac{\mu_o}{4\pi} I_1 \int_0^{\pi R} \frac{dl}{R^2} = \frac{\mu_o I_1}{4R} = \pi \mu I$$

(a) $B_{net} = \pi \mu I \quad \vec{\otimes}$

$$B_{"O"} = \frac{\mu_o I_2}{2\pi(0.9)} - \pi \times 10^{-6} = 0$$

(b) $I_2 = 7.85A$

Problem 4

(a) $\Phi = AB = 2ab e^{-0.5t}$

(b)
$$E_i = -\frac{d\Phi}{dt} = ab e^{-0.5t}$$

(c) Use Kirchhoff's Voltage rule

$$\varepsilon_i = \frac{q}{C} + I R$$

$$\frac{dq}{dt} + \frac{1}{RC} q = \frac{ab}{R} e^{-0.5t} \quad QED$$
