Electricity and Magnetism PHYCS 102
Final 2002 – 03

<u>Useful constants:</u>

Coulomb's constant, $k = 9 \times 10^9$ N.m^2/C^2

Permittivity of free space, $\varepsilon_o = 8.85 \times 10^{-12} C^2 / N.m^2$

Permeability of free space, $\mu_o = 4\pi \times 10^{-7} T.m/A$

Electron charge, $e = -1.6 \times 10^{-19} C$

Electron mass, $m_e = 9.11 \times 10^{-31} kg$

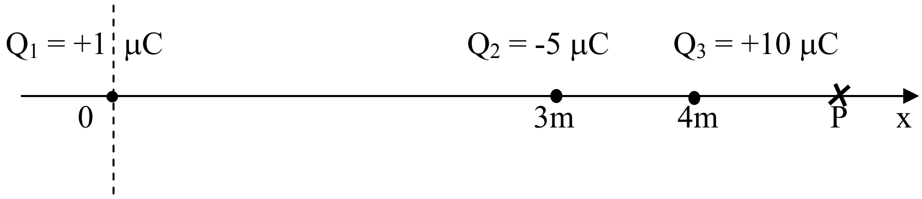
Acceleration due to gravity, $g = 9.8\ m/s^2$

Answer all problems in the exam copy book.

1- Three point charges are placed on the x-axis as shown in the figure.

a) Find the direction and magnitude of the electric field due to the three charges at point P located at x = 5m from origin.

b) Determine the magnitude and direction of the electric force experienced by Q_2 due to the other two charges.



<u>Solution:</u>

a) $$\vec{E}_P = K\left[\frac{Q_1}{5^2}\vec{i} + \frac{Q_2}{2^2}\left(-\vec{i}\right) + \frac{Q_3}{1^2}\vec{i}\right] = 7.91 \times 10^4 \frac{N}{C}\vec{i}$$

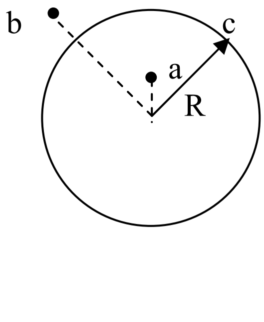
b) $$\vec{F}_{Q_2} = K\left[\frac{Q_1 Q_2}{3^2}\left(-\vec{i}\right) + \frac{Q_3 Q_2}{1^2}\vec{i}\right] = 0.445 N\vec{i}$$

2- An insulating solid sphere of radius R has a volume charge density ρ.

a) Determine the magnitude of the electric field at point **a**, which is at a distance r_a from the center.

b) Determine the magnitude of the electric field at point **b**, which is at a distance r_b from the center.

c) Find the potential V at the surface of the sphere (point **c**). Take V = 0 at infinity.



<u>Solution:</u>

a) Apply Gauss's Law

$$E_a\left(4\pi r_a^2\right)=\frac{\rho\left(\frac{4}{3}\pi r_a^3\right)}{\varepsilon_o} \quad , \quad E_a=\frac{\rho r_a}{3\varepsilon_o}$$

b) $$E_b\left(4\pi r_b^2\right)=\frac{\rho\left(\frac{4}{3}\pi R^3\right)}{\varepsilon_o} \quad , \quad E_b=\frac{\rho R^3}{3\varepsilon_o r^2}$$

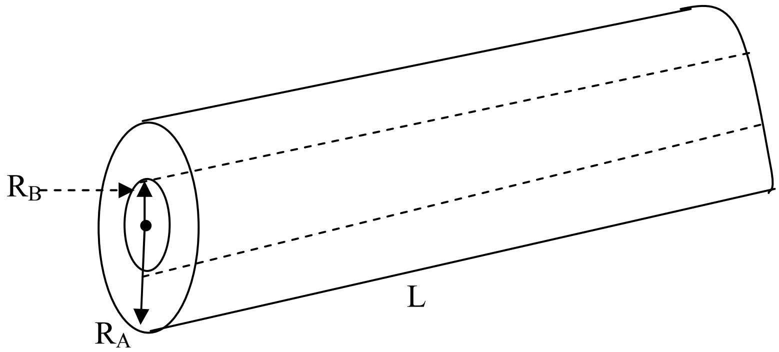
c) $$V_c-V_\infty=\int_R^\infty E(r)\,dr \quad , \quad V_c-V_\infty=\int\left[\frac{\rho R^3}{3\varepsilon_o r^2}\right]dr \quad , \quad V_c-V_\infty=\frac{\rho R^2}{3\varepsilon_o}$$

3- Two thin conducting cylindrical shells form a capacitor as shown in the figure. The outer shell has a radius R_A and carries a charge –Q, while the inner shell has a radius R_B and carries a charge +Q. The two coaxial shells have the same length L.

a) Show that the potential difference between the two shells is:

$$V_B-V_A=\left(\frac{Q}{2\pi\varepsilon_o L}\right) In\left(\frac{R_A}{R_B}\right)$$

b) Use the results of the previous part to determine the capacitance of this cylindrical capacitor.



<u>Solution:</u>

Consider a cylindrical Gaussian surface of length *l* and radius *r*:

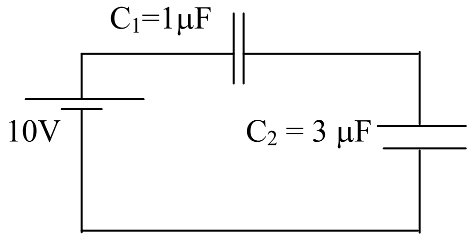
$$E\left(2\pi rl\right)=\frac{\lambda l}{\varepsilon_o} \quad , \quad E=\frac{\lambda}{2\pi\varepsilon_o r}=\frac{Q/L}{2\pi\varepsilon_o r} \quad , \quad V_B-V_A=\int_{R_B}^{R_A} E\,dr=\frac{Q}{2\pi\varepsilon_o L}\ell n\left(\frac{R_A}{R_B}\right)$$

$$C=\frac{Q}{V_{BA}}=\frac{2\pi\varepsilon_o L}{\ell n\left(\frac{R_A}{R_B}\right)}$$

4- A simple circuit consists of two capacitors and a battery as shown in the figure.

a) Determine the equivalent capacitance.

b) Now, while the battery remains connected, a dielectric with dielectric constant $\kappa = 2$, is inserted into (and completely fills) C_2. Determine the new equivalent capacitance.
c) Find the voltage across C_2 (after the dielectric has been inserted).



Solution:

a) $C_{eq} = \dfrac{C_1 C_2}{C_1 + C_2} = 0.75\mu F$

b) $C_{eq,N} = \dfrac{C_1 (kC_2)}{C_1 + kC_2} = 0.857\mu F$

c) $Q = V_{Battery}\ C_{eq,N} = 8.57\mu C$, $\Delta V_2 = \dfrac{Q}{kC_2} = \dfrac{8.57\mu C}{2\times 3\mu F} = 1.43V$

5- A copper wire 2 mm in radius and 50 cm in length, carries a current of 5A. If the number of free electrons per unit volume n = 8.5 x 10^{28} m^{-3}, and if the resistivity of the wire is 1.7 x 10^{-8} Ω.m, find:
a) The current density J.
b) The drift velocity v_d.
c) The electric field inside the wire.
d) The resistance of the wire.

Solution:

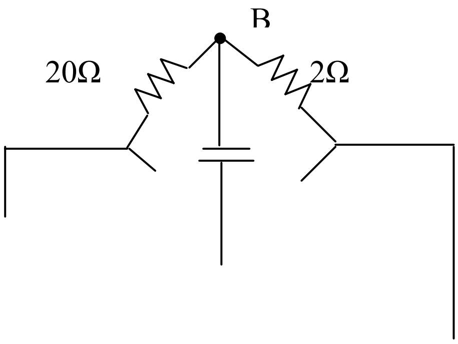
a) $J = \dfrac{5}{\pi r^2} = 3.97\times 10^5\ A/m^2$

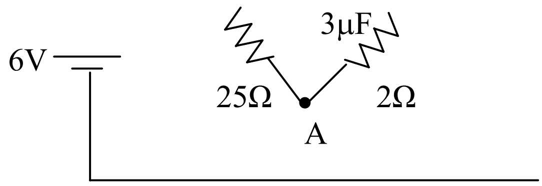
b) $J = nev_d$, $v_d = 0.029\ mm/s$

c) $E = \rho J = 6.74\times 10^{-3} V/m$

d) $R = \dfrac{\rho L}{A} = 6.76\times 10^{-4}\Omega$

6- Consider the circuit shown in the figure. Determine the voltage difference across the capacitor ($V_B - V_A$), assuming the circuit has been connected for a long time and the capacitor is fully charged.



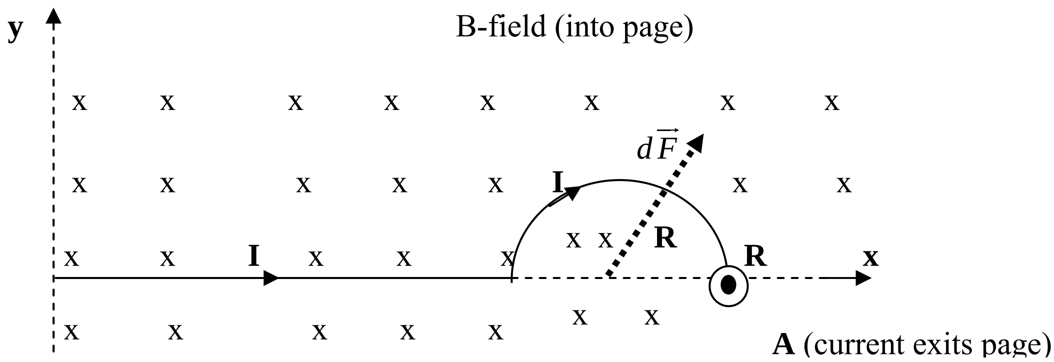


Solution:

$$R_{total} = \frac{(22)(27)}{22+27} = 12.12\Omega, \quad I_{main} = \frac{6}{12.12} = 0.5 = I_A + I_B,$$

$$22I_B = 27I_A \therefore I_A = 0.23A \quad and \quad I_B = 0.27A.$$
$$V_B - V_A = -20I_B + 25I_A = +0.35V$$

7- A conducting wire carrying a current, I, consists of three segments as shown in the figure; the first segment lies along the x-axis and has length L, the second segment is a semicircle of radius R, and the third segment is a straight line that exists the page at point A and extends for a length L. If the wire is subjected to a uniform magnetic field B (normal and entering into the page), determine the magnitude and direction of the net magnetic force experience by the wire.



Solution:
Third segment does not experience any force.

Seg.1 : $\vec{F}_1 = I\vec{L}_1 \wedge \vec{B} = (ILB)\vec{j}$

Seg.2: $\vec{F}_2 = I\int \vec{d\ell} \wedge \vec{B} = IB\int (d\ell) Sin\theta\,\vec{j}$

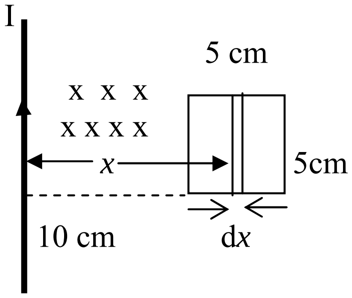
$= IB\int_o^{\pi} (Rd\theta) Sin\theta\,\vec{j} = 2IRB\vec{j}$

$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = IB(L+2R)\vec{j}$

8- An infinite straight wire carries a current I = 60 e^{-3t}. A conducting square loop of side 5 cm is placed at a distance of 10 cm from the wire as shown in the figure. Determine:

a) The magnitude of the magnetic flux through the loop.
b) The magnitude of the induced emf in the loop.

c) The magnitude and direction (clockwise or couter-clockwise) of the induced current in the loop if it has a resistance R = 5Ω



Solution:

a) $$\Phi = \int_{0.1m}^{0.15m} \left(\frac{\mu_o I}{2\pi x} \right) 0.05\, dx = \frac{\mu_o I (0.05)}{2\pi} \ell n \left(\frac{0.15}{0.1} \right) = 0.1m$$

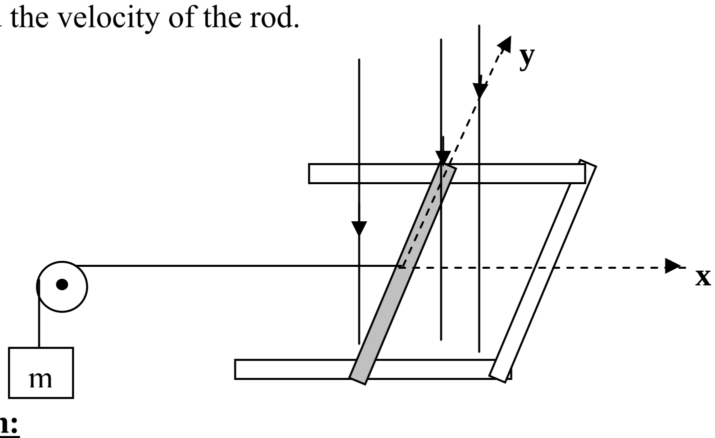
$$\Phi = \left(2.43 \times 10^{-7} \right) e^{-3t}\, Wb$$

b) $$\varepsilon = -\frac{d\Phi}{dt} = \left(7.3 \times 10^{-7} \right) e^{-3t} V$$

c) $$I = \frac{\varepsilon}{R} = \left(1.5 \times 10^{-7} \right) e^{-3t} A\ (clockwise)$$

9- A rod of length L =0.8 m and resistance 5 Ω is placed on a rail (with zero resistance) as shown in the figure. A magnetic field B = 2T is applied perpendicular to the plane of the rail and has the direction shown in the figure. The rod is attached to a mass m = 0.2 kg through a weightless chord that passes over a frictionless pulley. The rod moves to the left with constant velocity.

a) Determine the magnitude and direction (positive or negative y – axis) of the induced current in the rod.
b) Find the velocity of the rod.



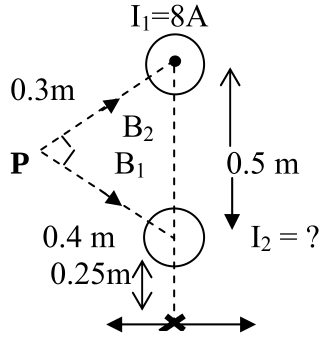
Solution:

a) $\sum F_x = 0$, $mg = I_i LB$; $I_i = 1.25A : (-y)$

b) $I_i = \frac{\varepsilon_i}{R} = \frac{BLv}{R}$ $\therefore$ $v = 3.9 m/s$

10- Tow long straight parallel wires are 0.5 m apart, as shown in the figure. The current $I_1 = 8A$ and is coming out of the page. Determine:

a) The magnitude and direction of the current I_2 given that the net magnetic field due to the two wires is zero at point S.
b) The net magnetic field (magnitude and direction) at point P.



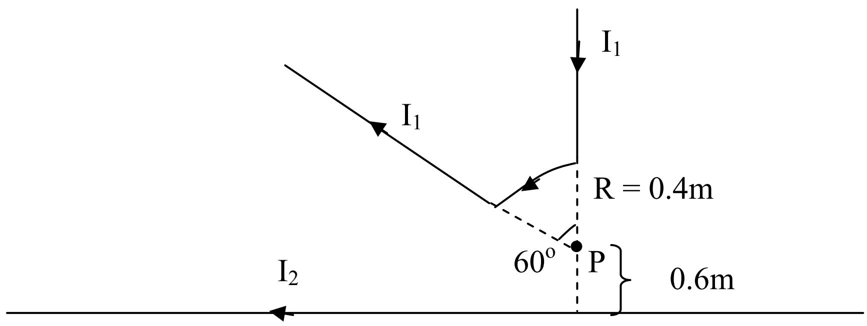
Solution:

a) $B_S = B_1 - B_2 = 0$ B2 S B1 $\frac{\quad}{0.75} - \frac{\quad}{0.25} = 0 \quad \therefore I_2 = 2.67A(\otimes)$

b) $B_\rho = \sqrt{B_1^2 + B_2^2} = \frac{\mu_o}{2\pi}\sqrt{\left(\frac{8}{0.3}\right)^2 + \left(\frac{2.67}{0.4}\right)^2} = 5.49\mu T$

11- The curved wire in the figure carries a current $I_1 = 4A$. It consists of two very long straight segments and an arc with radius R = 0.4 m that subtends an angle of 60°. A second infinite straight wire carries a current $I_2 = 2A$ and runs in the negative x – direction. Determine:

a) Magnitude and direction (into or out of page) of the magnetic field at point P due to I_1.
b) Magnitude and direction (into or out of page) of the magnetic field at point P due to I_2.
c) Magnitude and direction (into or out of page) of the resultant magnetic field at point P due to I_1 and I_2.



Solution:

a) $B_1 = \int \frac{\mu_o}{4\pi} I_1 \frac{d\ell}{R^2} = \frac{\mu_o I_1}{4\pi R^2} R\theta = 1.047\mu T(\square)$

b) $B_2 = \frac{\mu_o I_2}{2\pi(0.6)} = 0.67\mu T(\otimes)$

c) $B_{res} = B_1 - B_2 = 0.38\mu T(\square)$